DOES AMBIPOLAR DIFFUSION VIOLATE THE DEBYE SHIELDING REQUIREMENTS?

Debye shielding states that the distance over which a charge separation Δn can be sustained is

$$\lambda_D = \left(\frac{\varepsilon_o k T_e}{q^2 \Delta n}\right)^{1/2} \approx 750\left(\frac{T_e}{\Delta n}\right)^{1/2} \text{ cm}$$

where T_e (electron temperature) is in eV and Δn is in units of cm^{-3}. According to ambipolar diffusion theory, plasmas in containers actually are slightly electropositive. The slight positive space charge generates an electric field which accelerates ions out of the plasma and "holds back" electrons. This field (in plane parallel geometries) is

$$E_A = \frac{\pi}{L}\frac{kT_e}{q}\tan\left(\frac{\pi x}{L}\right)$$

where the separation between the plates is L. The implied charge separation can be obtained from applying Poisson's Equation $\bullet\, E = \frac{\rho}{\varepsilon_o}$.

$$\frac{dE_A}{dx} = \frac{q\Delta n}{\varepsilon_o} = \frac{d}{dx}\left(\frac{\pi}{L}\frac{kT_e}{q}\tan\left(\frac{\pi x}{L}\right)\right) = \left(\frac{\pi}{L}\right)^2 \frac{kT_e}{q}\frac{1}{\cos^2\left(\frac{\pi x}{L}\right)}$$

Solving for Δn we have

$$\Delta n(x) = \frac{\varepsilon_o k T_e}{q^2}\left(\frac{\pi}{L}\right)^2 \frac{1}{\cos^2\left(\frac{\pi x}{L}\right)}$$

Using typical values ($L = 2$ cm, $kT_e = 1$ eV) we have

$$\Delta n(x) = \frac{10^6 \text{cm}^{-3}}{\cos^2\left(\frac{\pi x}{L}\right)} = \frac{\Delta n_o}{\cos^2\left(\frac{\pi x}{L}\right)}$$

The charge separation (or depletion of negative charge) is small and increases towards the wall where the E-field also increases. If we solve for $\frac{L}{\pi}$, the diffusion length, we get

$$\frac{L}{\pi} = \left(\frac{1}{\cos\left(\frac{\pi x}{L}\right)}\right) \bullet \left(\frac{\varepsilon_o k T_e}{q^2 \Delta n(x)}\right)^{1/2} = \left(\frac{\varepsilon_o k T_e}{q^2 \Delta n_o}\right)^{1/2} = \lambda_o$$

where λ_o is the Debye length based on Δn_o. So we see that the charge separation dictated by ambipolar diffusion over the diffusion length of the container is consistent with a Debye length of the same distance. (Remember that λ_D is really based on maximum charge separation and not maximum or total density.)