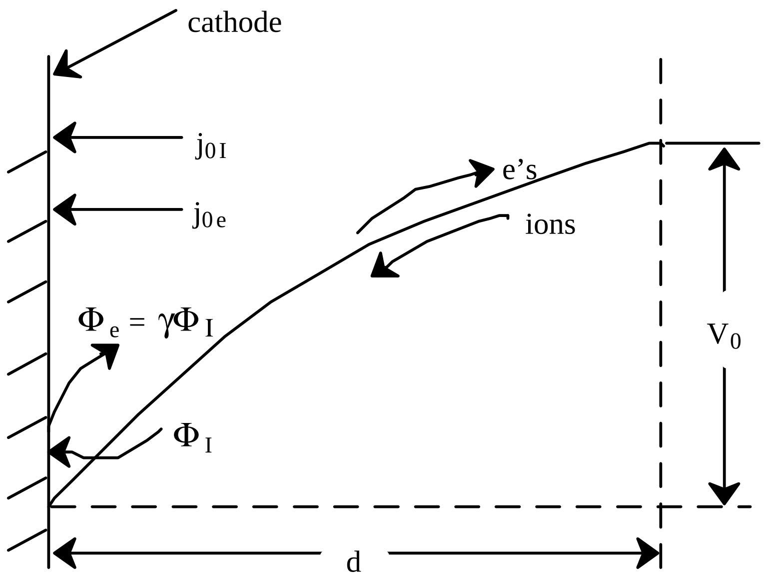
ANALYSIS OF THE CATHODE FALL OF A NORMAL GLOW



j_{0I} = current density of ions at cathode (A/cm^2)

j_{0e} = current density of e's at cathode (A/cm^2)

V_0 = voltage drop across cathode fall

d = thickness of cathode fall (cm)

Assume $n_e << N_I$ and the $N_I \cong$ constant in the cathode fall. Poisson's equation becomes

$$\frac{\partial E}{\partial x} = \frac{\rho}{\varepsilon_0} = \frac{q(N_I - n_e)}{\varepsilon_0} \approx \frac{qN_I}{\varepsilon_0} \approx \frac{j_I}{\varepsilon_0 v_I} \approx \text{constant}$$

$$j_I = q \cdot v_{drift} \cdot N_I \qquad \text{so that} \qquad N_I = \frac{j_I}{qv_I}$$

Integrating, $E(x) = E_0\left(1 - \frac{x}{d}\right)$ E_0 is electric field at the cathode where E_0 >> E in positive column. The cathode fall voltage drop is $V_0 = \int_0^d -E_0\left(1 - \frac{x}{d}\right)$.

$$V(x) = -E_0\left(x - \frac{x^2}{2d}\right), \quad V_0 = \frac{-E_0 d}{2}, \quad E_0 = \frac{-2V_0}{d}$$

Ions striking the cathode produce electrons by secondary emission with probability γ. The secondary electrons are accelerated back into the plasma producing ionizations in the dark space with Townsend coefficient α.

$$j_0 = j_{0I} + j_{0e} = j_{0I} + \gamma j_{0I} = (1 + \gamma) j_{0I}$$

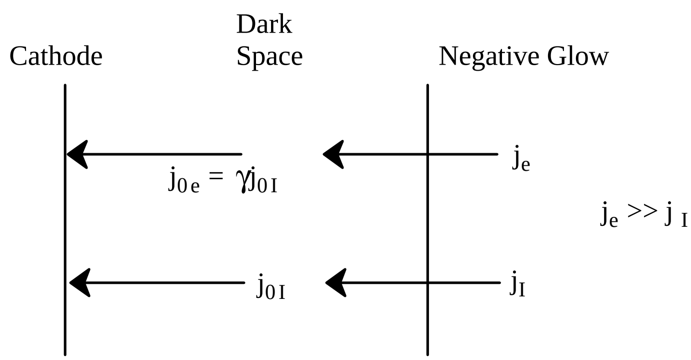
With $j_{0I} = qN_I v_I$, $v_I = \mu_I E_0$,

$$\left.\frac{\partial E}{\partial x}\right|_{x=0} = \frac{E_0}{d} = \frac{qN_I}{\varepsilon_0}, \qquad N_I = \frac{\varepsilon_0 E_0}{dq} \qquad \text{so} \qquad j_{0I} = q \cdot \left(\frac{\varepsilon_0 E_0}{dq}\right) \cdot \mu_I E_0 = \frac{\varepsilon_0 \mu_I E_0^2}{d}$$

The total current density is then

$$j_0 = (1+\gamma)\, j_{0I} = (1+\gamma)\frac{\varepsilon_0 \mu_I E_0^2}{d} = \frac{(1+\gamma) 4\varepsilon_0 \mu_I V_0^2}{d^3}$$

Define j_e = electron current at edge of dark space



Since $j_e >> j_I$ then

$$j_0 = j_{0e} + j_{0I} = j_e + j_I \approx j_e, \qquad j_{0I} = j_0 - j_{0e} \approx j_e - j_{0e}$$

But $j_{0e} = \gamma j_{0I} = \gamma(j_0 - j_{0e}) \approx \gamma(j_e - j_{0e})$

$$j_{0e} \approx \left(\frac{\gamma}{\gamma+1}\right) j_e$$

The electron current from the cathode through the dark space is amplified by electron impact ionization

$$\frac{dj}{dx} = j \cdot \alpha(x), \qquad \alpha = \text{First Townsend Coefficient}$$

$$j_e = j_{0e} \exp\left(\int_0^d \alpha(x)\,dx\right)$$

$$= j_{0e} \exp\left(\int_0^d \alpha\left(\frac{E}{N}(x)\right) dx\right)$$

With $j_{0e} \approx \left(\frac{\gamma}{\gamma+1}\right) j_e \approx \left(\frac{\gamma}{\gamma+1}\right) j_{0e} \exp\left(\int_0^d \alpha(x)\,dx\right)$

$$\ln\left(1+\frac{1}{\gamma}\right) = \int_0^d \alpha\left(\frac{E}{N}(x)\right)dx$$

where $E(x) = E_0\left(1-\frac{x}{d}\right)$, $E_0 = \frac{-2V_0}{d}$

From empirical data,

$$\alpha = A \cdot p \cdot \exp\left(\frac{-Bp}{E}\right)$$

p = pressure (Torr), $A = \frac{1}{\text{cm - Torr}}$, $B = \frac{V}{\text{cm - Torr}}$

So, $$\ln\left(1+\frac{1}{\gamma}\right) = \int_0^d Ap\,\exp\left(\frac{-Bp}{E_0\left(1-\frac{x}{d}\right)}\right)dx$$

The integral can be solved analytically

$$\ln\left(1+\frac{1}{\gamma}\right) = \left[\frac{A(pd)^2 B}{2V_0}\right] S\left(\frac{2V_0}{(pd)B}\right)$$

where $S(x) = \int_0^x e^{-\frac{1}{y}}dy = xe^{-\frac{1}{x}} - E_1\left(\frac{1}{x}\right)$ $E_1 =$ Exponential Integral.

We can now solve for V_0 in terms of d. A second relationship between v_0 and d is

$$j_0 = \frac{4\varepsilon_0 V_0^2 \mu_I (1+\gamma)}{d^3}$$

We could in principle solve for two of j_0, V_0 and d as a function of the third. Up to now, there is nothing unique to the normal glow.

Define two functions

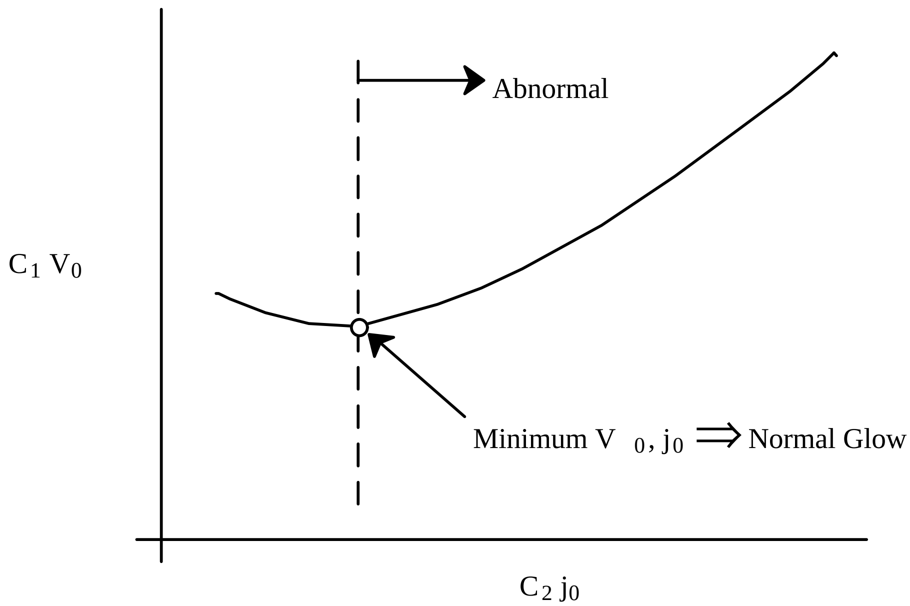
$$C_1 = \frac{2A}{B\ \ln\left(1+\frac{1}{\gamma}\right)}, \qquad C_2 = \frac{\ln\left(1+\frac{1}{\gamma}\right)}{\varepsilon_0 A B^2 p^2 (p \cdot \mu_I)(1+\gamma)}$$

where
$$1 = \frac{(C_1V_0)^{1/3}}{(C_2 j_0)^{2/3}} \cdot S\left[\left(\frac{C_1V_0}{C_2 j_0}\right)^{1/3}\right]$$

Note: Since $\mu_I \sim \frac{1}{\nu_I} \sim \frac{1}{p}$, then $p\mu_I$ is a constant.

If μ_0 is the mobility at pressure p_0, $\mu_I = \mu_0 \frac{p_0}{p}$.

Plot all points satisfying relationship



Minimum occurs at: $C_1 V_{NORMAL} = 6.0$

$C_2 j_n = 0.67$

which yields

$$V_{normal} = \frac{3B}{A}\ln\left(1+\frac{1}{\gamma}\right) \equiv \text{ function of gas, metal but not pressure}$$

$$j_n = \left[5.92 \times 10^{-14}\right] \cdot \frac{AB^2(\mu_0 p_0)(1+\gamma)}{\ln\left(1+\frac{1}{\gamma}\right)} \cdot p^2$$

where $A \equiv \frac{1}{\text{cm} - \text{Torr}}, B \equiv \frac{V}{\text{cm} - \text{Torr}}, j_n \equiv \frac{A}{\text{cm}^2}$

$p \equiv \text{Torr}$, $\mu_0 \equiv \frac{\text{cm}^2}{\text{v} - \text{s}}$ is the mobility at p_0

Note that j_n ~ [function of gas type, metal] · $p^2 \sim p^2$

Combine to find $d_n p = \frac{0.82}{A} \ln\left(1 + \frac{1}{\gamma}\right) \equiv$ function gas type and metal

Typical values are $V_n \approx$ 100-300 V, $d_n p \approx$ 0.25-2.5 Torr-cm

$$\frac{j_n}{p^2} \approx 0.005 - 0.5 \frac{\text{mA}}{\text{cm}^2\ \text{Torr}^2}$$