ANALYTIC EXPRESSIONS FOR ELECTRON IMPACT CROSS SECTIONS

$m_e = electron\ mass = 9.11\times10^{-28}\,g,\quad M = atomic\ mass,\ \ q = elementary\ charge,$

$a = Bohr\ radius = \dfrac{\hbar^2}{m_e q^2} = 0.529\times10^{-8}\,cm,\quad \hbar = \dfrac{Planck\ constant}{2\pi} = \dfrac{6.63\times10^{-34}\,J-s}{2\pi},$

$\varepsilon = electron\ energy,\quad \mu = reduced\ mass = \dfrac{m_e M}{(m_e + M)},\quad k = wave\ vector = \left(\dfrac{2\varepsilon m_e}{\hbar^2}\right)^{1/2},$

$\lambda_D = Debye\ Length = \left(\dfrac{\varepsilon_o k_B T_e}{m_e n_e}\right)^{1/2},\quad k_B = Boltzmann's\ constant = 1.38\times10^{-23}\,J-K,$

$\varepsilon_o = permittivity\ free\ space = 8.85\times10^{-12}\,\dfrac{F}{m},\quad n_e = electron\ density,\quad N_I = ion\ density,$

$T_e = electron\ temperature,\quad Z = charge\ number\ of\ ion,\quad f_{ij} = oscillator\ strength,$

$b_o = scattering\ parameter\ for\ 90^0\ collision = \dfrac{Zq^2}{8\pi\varepsilon_o},\quad E_H = Rydberg\ energy = 13.6\ eV,$

ξ_n = *number of orbital electrons in the outer shell of the atom*

<u>Born cross section for elastic collisions:</u>

$$\sigma(\varepsilon) = \frac{16\pi\mu^2 q^4 a^4}{\hbar^4\left(4k^2a^2+1\right)}$$

For electron scattering on helium, $\sigma(\varepsilon) = \dfrac{14\times10^{-16}}{0.3\cdot\varepsilon(eV)+1}\,cm^2$

<u>Coulomb cross section for electron-ion collisions:</u>

$$\sigma(\varepsilon) = 4\pi b_o^2\left[\ln\left(1+\left(\frac{\lambda_D}{b_o}\right)^2\right)\right]$$

Electron-ion collision frequency for a Maxwellian electron energy distribution

$$\nu_{eI} = \frac{4\sqrt{2\pi}}{3}\left(\frac{m_e}{k_B T_e}\right)^{3/2}\left(\frac{q^2}{4\pi\varepsilon_o m_e}\right)^{3/2}\ln\Lambda \approx N_I\left(cm^{-3}\right)\frac{2.9\times10^{-6}}{T_e(eV)}\ln\Lambda\, s^{-1}$$

$$\Lambda = \frac{\lambda_D}{\overline{b}_o},\ \ \overline{b}_o = \frac{Zq^2}{12\pi\varepsilon_o k_B T_e}$$

Drawin expression for Born-Bethe electron impact excitation cross section:

For excitation of state j (higher energy) from state i (lower energy) having energy separation E_{ij}

$$\sigma_{ij}(\varepsilon) = 4\pi a^2 \left(\frac{E_H}{E_{ij}} \right) f_{ij} \alpha_{ij} \frac{U_{ij} - 1}{U_{ij}^2} \ln(1.25 \beta_{ij} U_{ij}), \quad U_{ij} = \frac{\varepsilon}{E_{ij}}$$

α_{ij} and β_{ij} are atomic constants depending on the i,j with values of order 1.

Drawin expression for electron-impact ionization:

For ionization of atom or molecule with ionization potential E_I

$$\sigma_I(\varepsilon) = 2.66\pi a^2 \xi_n \frac{U-1}{U^2} \ln(1.25 \beta U), \quad U = \frac{\varepsilon}{E_I}, \quad \beta = 1 + \left(\frac{Z_e - 1}{Z_e + 2} \right), \quad Z_e = \left(\frac{E_I}{E_H} \right)^{1/2}$$

Gryzinski expression for electron-impact ionization:

For ionization of atom or molecule with ionization potential E_I

$$\sigma_I(\varepsilon) = 4\pi a^2 \xi_n \left(\frac{E_H}{E_I} \right)^2 g(U),$$

$$g(U) = \frac{1}{U} \left(\frac{U-1}{U+1} \right)^{3/2} \left(1 + \frac{2}{3} \left(1 - \frac{1}{2U} \right) \ln\left(2.7 + (U-1)^{1/2}\right) \right), \quad U = \frac{\varepsilon}{E_I}, \quad \beta = 1 + \left(\frac{Z_e - 1}{Z_e + 2} \right), \quad Z_e = \left(\frac{E_I}{E_H} \right)^{1/2}$$

Super-elastic collisions:

For de-excitation of state j (higher energy) to state i (lower energy) having energy separation E_{ij}

$$\sigma_{ji}(\varepsilon) = \frac{g_i}{g_j} \left(\frac{\varepsilon + E_{ij}}{\varepsilon} \right) \sigma_{ji}(\varepsilon + E_{ij})$$

$\sigma_{ij}(\varepsilon)$ is the excitation cross section of state j (higher energy) from state i (lower energy)
g_j is the degeneracy of state j.