

EXAMPLE: CYLINDRICAL GLOW DISCHARGE PARAMETERS

What are the electron density, electron temperature and self-sustaining electric field in a low temperature plasma having the following properties?

$$j = 10 \frac{\text{mA}}{\text{cm}^2}, \quad \text{cylindrical tube 1.5 cm diameter}$$

$$P = 3 \text{ Torr}, \quad T_g = 400 \text{ K}, \quad M_g = 40 \text{ AMU}, \quad D_{\text{ION}} = 100 \frac{\text{cm}^2}{\text{s}}$$

$$k_m = 3 \times 10^{-7} \frac{\text{cm}^3}{\text{s}}, \quad k_{\text{ion}} = 3 \times 10^{-11} \exp\left(\frac{-\Delta\epsilon}{T_e}\right) \frac{\text{cm}^3}{\text{s}}, \quad \Delta\epsilon = 9 \text{ eV}$$

ELECTRON TEMPERATURE (assume diffusion losses dominate):

$$\frac{\partial n_e}{\partial t} = 0 = n_e \cdot k_{\text{ION}} \cdot N_G - \frac{D_A N^+}{\Lambda^2} = 0$$

$$n_e \approx N^+, \quad \Lambda = \frac{R}{2.405}, \quad D_A = D_I \left(1 + \frac{T_e}{T_I}\right), \quad T_I \approx T_g$$

$$N_G = \frac{9.654 \times 10^{18} \times 3 \text{ Torr}}{400 \text{ K}} = 7.24 \times 10^{16} \text{ cm}^{-3}$$

$$\begin{aligned} 3 \times 10^{-11} \exp\left(\frac{-9 \text{ eV}}{T_e}\right) \frac{\text{cm}^3}{\text{s}} \cdot 7.24 \times 10^{16} \text{ cm}^{-3} \\ = 100 \frac{\text{cm}^2}{\text{s}} \times \frac{(2.405)^2}{(1.5/2)^2 \text{ cm}^2} \left(1 + \frac{T_e}{(400/11594.2) \text{ eV}}\right) \end{aligned}$$

Solve for T_e and find $T_e = 2.76 \text{ eV}$.

AXIAL ELECTRIC FIELD:

$$T_e = T_g + \frac{2}{3k_B} \frac{1}{\left(\frac{2m_e}{M}\right)} \left(\frac{q^2}{m_e k_m^2} \left(\frac{E}{N}\right)^2 - \frac{k_I \Delta\epsilon}{k_m} \right)$$

$$\left(\frac{E}{N}\right) = \left[\left(\left(\frac{2m_e}{M} \right) \frac{3}{2} k_B (T_e - T_g) + \frac{k_I \Delta\epsilon}{k_m} \right) \frac{m_e k_m^2}{q^2} \right]^{1/2}$$

$$= \left[\left(\left(\frac{2 \times 0.911 \times 10^{-27} \text{ g}}{40 \times 1.67 \times 10^{-24} \text{ g}} \right) \times \frac{3}{2} \left(2.76 - \frac{400}{11594.2} \right) \text{ eV} + \frac{3 \times 10^{-11} \exp\left(\frac{-9}{2.76}\right) \cdot 9 \text{ eV}}{3 \times 10^{-7}} \right) \right.$$

$$\left. \cdot 1.6 \times 10^{-19} \frac{\text{J}}{\text{eV}} \cdot \frac{0.911 \times 10^{-30} \text{ kg}}{(1.6 \times 10^{-19} \text{ c})^2} \times \left(3 \times 10^{-7} \frac{\text{cm}^3}{\text{s}} \times 10^{-6} \frac{\text{m}^3}{\text{cm}^3} \right)^2 \right]^{1/2}$$

$$= 8.65 \times 10^{-21} \frac{\text{J}}{\text{c}} \cdot \text{m}^2 = 8.65 \times 10^{-21} \text{ V} \cdot \text{m}^2 \times \left(\frac{10^4 \text{ cm}^2}{\text{m}^2} \right)$$

$$\left(\frac{\text{E}}{\text{N}} \right) = 8.65 \times 10^{-17} \text{ V} \cdot \text{cm}^2 = 8.65 \text{ Td}$$

$$E = \left(\frac{\text{E}}{\text{N}} \right) \text{ N} = 8.65 \times 10^{-17} \cdot 7.2 \times 10^{16} \text{ V} \cdot \text{cm}^2 \times \frac{1}{\text{cm}^3} = 6.2 \frac{\text{V}}{\text{cm}}$$

ELECTRON DENSITY:

$$j = \sigma \cdot E = q \cdot n_e \cdot \mu \cdot E = \frac{q^2 \cdot n_e \cdot q}{m_e v_m} \cdot E = \frac{q^2 \cdot n_e}{m_e \cdot k_m} \cdot \left(\frac{\text{E}}{\text{N}} \right)$$

$$n_e = \frac{j \cdot m_e \cdot k_m}{q^2 \cdot \left(\frac{\text{E}}{\text{N}} \right)}$$

$$= \frac{10 \times 10^{-3} \text{ c} / (\text{cm}^2 \cdot \text{s}) \cdot 0.911 \times 10^{-30} \text{ kg} \times 3 \times 10^{-7} \frac{\text{cm}^3}{\text{s}}}{(1.6 \times 10^{-19} \text{ c})^2 \times 8.65 \times 10^{-17} \frac{\text{kg} \cdot \text{m}^2 \cdot \text{cm}^2}{\text{s}^2 \cdot \text{cm c}} \times 10^4 \frac{\text{cm}^2}{\text{m}^2}}$$

$$= 1.23 \times 10^{11} \text{ cm}^{-3}$$