DERIVATION OF EINSTEIN RELATION

In equilibrium, the density of particles having temperature T in an electric potential U is

$$N = N_o \exp\left(\frac{qU}{kT}\right), \quad q = \pm e$$

where k = Boltzmann's constant. The gradient of particles due to a gradient in potential is

$$\nabla N = \frac{q}{kT} \nabla U \cdot \left(N_o \exp\left(\frac{qU}{kT}\right)\right) = \frac{q}{kT} \nabla U \cdot N$$

where the Electric field is $-\nabla U$. The total flux of particles at equilibrium is zero, and is

$$j = \pm \mu \nabla U \cdot N - D \nabla N = 0$$

$$= \mu \nabla U \cdot N - D \frac{q}{kT} \nabla U N$$

$$= \pm \nabla U \cdot N \left(\mu - \frac{De}{kT}\right) = 0$$

$$D = \frac{kT\mu}{e}$$

where μ is the mobility $\left(\frac{cm^2}{V-s}\right)$ and D is the diffusion coefficient $\frac{cm^2}{s}$.