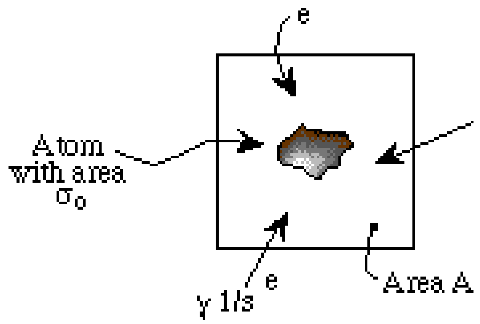
RELATION BETWEEN CROSS SECTION, FLUX, RATE OF COLLISION, AND RATE COEFFICIENT

Consider a single atom with cross sectional "area" of σ_o cm^2. If electrons with energy ε randomly pass through a plane of area A, then the probability of any one single electron passing through area A of hitting the atom is



$$P_1 = \frac{\sigma_o}{A}$$

Now if the total number of electrons with energy ε passing through A per second is γ, then the rate of all electrons with energy ε hitting a single atom is

$$R_1 = \gamma P_1 \frac{1}{s} = \gamma \frac{\sigma_o}{A} = \left(\frac{\gamma}{A}\right) \sigma_o = \varphi \sigma_o \frac{1}{s}$$

where the electron flux $\varphi \left[\frac{}{\text{cm}^2\text{-s}}\right]$ is the number of electrons passing through the plane per unit area.

Now if there are N_T total atoms in a volume V then the total rate of collisions of all electrons striking all atoms per unit volume is

$$R_2 = \left(R_1 \frac{N_T}{V}\right) \frac{1}{\text{cm}^3\text{-s}} = R_1 N \frac{1}{\text{cm}^3\text{-s}}, \quad N = \frac{N_T}{V} \frac{1}{\text{cm}^3}$$

$$= \varphi \sigma_o N \frac{1}{\text{cm}^3\text{-s}}$$

where N is the number density of atoms.

So finally the rate at which a single electron with energy ε strikes a single atom per unit volume (n_T total electrons with energy ε in V) is

$$R_3 = R_2 \frac{1}{\left(\frac{n_T}{V}\right)\left(\frac{N_T}{V}\right)} \frac{\text{cm}^3}{s} = \frac{\varphi \sigma_o N}{nN} = \frac{\varphi \sigma_o}{n} \frac{\text{cm}^3}{s},$$

where n_T is the total number of electrons in volume V having energy ε, and where $n = \frac{n_T}{V}$ $\left(\frac{\ }{\mathrm{cm}^3}\right)$ is the number density of electrons at energy ε.

The flux of electrons at energy ε is

$$\varphi(\varepsilon) = n(\varepsilon)\left(\frac{2\varepsilon}{m_e}\right)^{1/2} = n(\varepsilon)v\,\frac{\#}{\mathrm{cm}^2\text{-s}}$$

where $v = \left(\frac{2\varepsilon}{m_e}\right)^{1/2}$ is the electron velocity, so

$$R_3(\varepsilon) = \frac{\varphi(\varepsilon)\,\sigma_o(\varepsilon)}{n(\varepsilon)} = \sigma_o(\varepsilon)v\,\frac{\mathrm{cm}^3}{\mathrm{s}}\,.$$

With these definitions, the average rate at which a single electron strikes a single atom per unit volume for an entire distribution of electron energies is

$$k = \int_0^\infty f(\varepsilon)R_3(\varepsilon)\,d\varepsilon = \int_0^\infty f(\varepsilon)\,\sigma_o(\varepsilon)\left(\frac{2\varepsilon}{m_e}\right)^{1/2} d\varepsilon\ \frac{\mathrm{cm}^3}{\mathrm{s}}$$
$$= \langle \sigma_o\, v \rangle$$

where k = electron impact rate coefficient $\left(\frac{\mathrm{cm}^3}{\mathrm{s}}\right)$

f(ε) = electron energy distribution function (eV^{-1}) normalized as

$$\int_0^\infty f(\varepsilon)\,d\varepsilon = 1$$

Example: The time rate of change of electron density due to electron impact ionization collisions is

$$\frac{dn_e}{dt}\left(\frac{1}{\mathrm{cm}^3\text{-s}}\right) = \begin{bmatrix}\text{Rate of ionizing}\\ \text{collisions per unit}\\ \text{volume of a single}\\ \text{electron with a}\\ \text{single atom}\end{bmatrix} \times \begin{bmatrix}\text{Number of}\\ \text{electrons}\\ \text{per unit volume}\end{bmatrix} \times \begin{bmatrix}\text{Number of}\\ \text{atoms}\\ \text{per unit volume}\end{bmatrix}$$

$$k_I\left(\frac{\mathrm{cm}^3}{\mathrm{s}}\right) \qquad n_e\left(\frac{1}{\mathrm{cm}^3}\right) \qquad N\left(\frac{1}{\mathrm{cm}^3}\right)$$

$$= n_e\, k_I\, N\left(\frac{1}{\mathrm{cm}^3\text{-s}}\right)$$