$\underline{f(\vec{v})\ \text{vs}\ f(v)\ \text{vs}\ f(\varepsilon)}$

$f(\vec{v})$ has normalization

$$\int f_o(\vec{v})\, d^3v = 1, \qquad f_o(\vec{v}) \equiv eV^{-3/2}$$

When using the Spherical Harmonic Expansion, $f_o(\vec{v}) \rightarrow f_o(v)$ (function of speed v instead of velocity $\vec{v}$). The normalization is then

$$\int f_o(v)\, 4\pi v^2 dv, \qquad f_o(v) \equiv eV^{-3/2}$$

where 4 v ^{2}dv is the volume element (a spherical shell in velocity space). Once we have f(v) we can equivalently express the distribution as f(ε), where

$$\frac{1}{2}\, m_e\, v^2 = \varepsilon$$

$$m_e v\, dv = d\varepsilon, \qquad dv = \frac{d\varepsilon}{m_e v} = \frac{d\varepsilon}{(2 m_e \varepsilon)^{1/2}}, \qquad v = \left(\frac{2\varepsilon}{m_e}\right)^{1/2}$$

Therefore

$$f_o(\varepsilon)\, d\varepsilon = f_o(v)\, d^3v$$

Normalization eV^{-1}: $\quad f_o(\varepsilon)\ eV^{-1} = f_o(v)\, \dfrac{4\pi v^2 dv}{d\varepsilon} = \dfrac{4\pi\sqrt{2}}{m_e^{3/2}}\, \varepsilon^{1/2}\, f_o(v)$

or

Normalization $eV^{-3/2}$: $\quad f_o(\varepsilon)\ eV^{-3/2} = \dfrac{4\pi\sqrt{2}}{m_e^{3/2}}\, f_o(v)$

Boltzmann's equation $\left(\dfrac{\partial}{\partial z} = \dfrac{\partial}{\partial t} = 0,\ \text{elastic collisions only}\right)$ in an energy representation is then

$$\frac{\partial}{\partial \varepsilon}\left(\frac{q^2E^2\varepsilon}{3N\,\sigma_m(\varepsilon)}\left(\frac{\partial f_o(\varepsilon)}{\partial \varepsilon}\right)\right) + \frac{2m_e}{M}\frac{\partial}{\partial \varepsilon}\left[\varepsilon^2 N\,\sigma_m(\varepsilon)\left[f_o(\varepsilon) + kT_g\left(\frac{\partial f_o}{\partial \varepsilon}\right)\right]\right] = 0$$

where $f_o(\varepsilon) = eV^{-1}$.