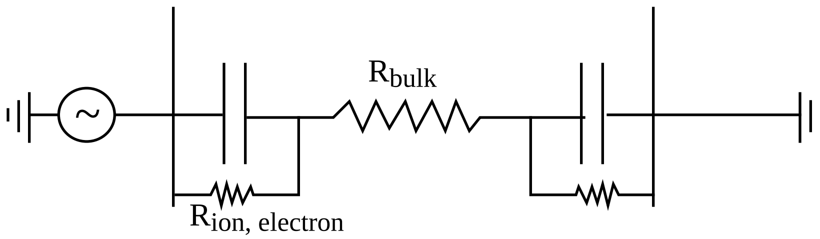
B. Simple Power Balance Model for RF Discharges

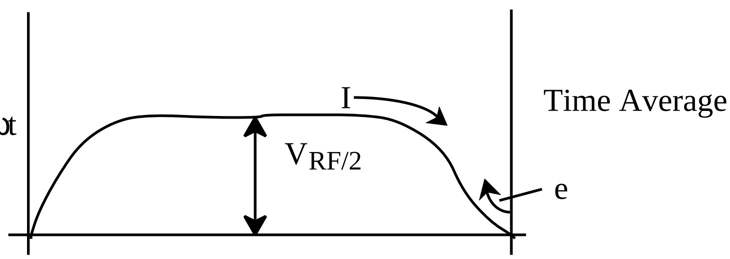
1. Assume $V_{sheaths} > V_{bulk}$. We control j_0, ω, dimensions

2. Power deposition is by

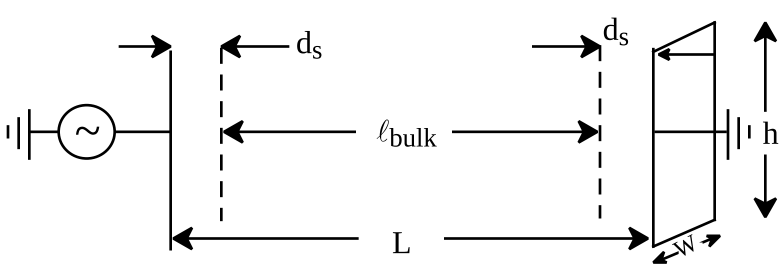
a. Electron heating in the bulk
b. Ion bombardment of electrodes
c. Secondary electron emission with acceleration back into the plasma
d. Stochastic heating



$V = V_{RF} \sin \omega t$
$I = j \cdot A$
$j = j_0 \cos \omega t$



3. Current through sheaths is dominantly capacitive with



4. Bulk electron heating: P_B (Watts)

$$P_B = \int < j \; E >_{RMS} \; dV_{bulk} = \int \left(\frac{j^2}{\sigma} \right) dV_{bulk}, \quad \sigma = \text{conductivity}$$

$$= \frac{1}{2} \, j_0^2 \left(\frac{m_e \nu_m}{q^2 n_e} \right) l_B \; h \; w$$

5. Ion acceleration: P_I

a. Assume that there is no recombination or attachment in the bulk plasma. ($n_e = n_I$)
b. Every ion produced in the bulk is lost to the electrodes.
c. Ions enter the sheath with the Bohm speed.

$$\int (n_e \; k_{ion} \; N_{gas}) \; dV_{bulk} = \oint_{sheath} n_I \; u_{Bohm} \; dA_{electrodes}$$

$$n_e \, k_{ion} \, N_{gas} \, l_B \cdot h \cdot w = n_I \, u_{Bohm} \cdot w \cdot h \cdot 2$$

$$P_I = n_I \cdot u_{Bohm} \cdot w \cdot h \cdot \left(\frac{qV_{RF}}{2} + \frac{qkT_e}{2} \right) \cdot 2$$

Where the first term is the ion acceleration in the sheath, the second term is the Bohm speed contribution, and the last “2” is for 2 electrodes.

6. Secondary Electron Power: P_e

For every ion striking the electrode we get γ electrons which accelerate back into the plasma.

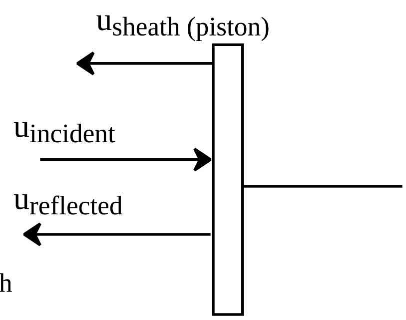
$$P_e = \gamma \left(\frac{qV_{RF}}{2} \right) \cdot n_I \cdot u_{Bohm} \cdot w \cdot h \cdot 2$$

where the last “2” is for two electrodes.

7. Stochastic electron heating: P_s

a. Electrons are accelerating by advancing sheath in the same manner as an elastic collision of a ball from a piston.

b.



$u_{refl} = -u_{inc} + 2u_{sheath}$

The power into the electrons for 2 sheaths is then

$$P_s = 2\int_{u_{sh}}^{\infty} \underbrace{\frac{1}{2} m_e\left(u_{refl}^2 - u_{inc}^2\right)}_{\text{energy gain}} \ \underbrace{\left(u_{inc} - u_{sh}\right)}_{\text{rate of collision}} \ n_e f(u_{inc})\ du_{inc} \ \ w \ \ h$$

$$= -2 \ \ 2m_e \int_{u_{sh}}^{\infty} u_{sh}\left(u_{inc} - u_{sh}\right)^2 n_e f(u_{inc})\ du_{inc} \ \ w \ \ h$$

Assume that $u_{sheath} = u_0 \sin \omega t$ and integrate over time

$$P_s \approx 2 \cdot 2m_e n_e\, u_0^2 \int_{u_{sh}}^{\infty} u_{inc}\ f(u_{inc})\, du_{inc} \ \ w \ \ h$$

Since $u_{sh} << \langle u_{inc} \rangle$, then set lower limit to zero. Note that

$$\int_0^{\infty} u_{inc} f(u_{inc})\, du_{inc} = \frac{\bar{v}_{th}}{4}$$

so

$$P_s = 2 \cdot \frac{m_e n_e\ u_0^2}{2}\ v_{th} \cdot w \cdot h$$

The conduction current in the bulk plasma must equal the rate at which the electrons are swept out by the sheath, so

$$j_0 = q \cdot n_e \cdot u_0$$

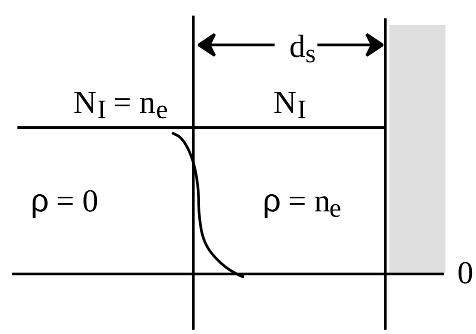
$$P_s = \frac{m_e v_{th}}{q^2 n_e} j_0^2 \cdot w \cdot h$$

8. The total power deposition is then:

$$\begin{aligned} P_{total} &= P_{bulk} + P_{e-sec} + P_{ion} + P_{stochastic} \\ &= \frac{1}{2} j_0^2 \left(\frac{m_e \nu_m}{q^2 n_e} \right) l_B \; h \; w \\ &+ 2 \; n_e \; u_{Bohm} \left(\frac{qV_{RF}}{2} (1 + \gamma) + \frac{kT_e}{2} \right) h \; w \\ &+ \frac{m_e v_{th}}{q^2 n_e} j_0^2 \; h \; w \end{aligned}$$

9. To complete the analysis, we need a relationship between V_{RF} and j_0, n_e and T_e.

V_{RF} If we assume that the majority of the applied voltage is across the sheaths, then on the average



$$V_{RF} \approx \int_0^{d_s} E \; dx = -\frac{qn_e \; d_s^2}{2\varepsilon_0}$$

Since the capacitive current through the sheath is

$$I = C_s \frac{dV_{RF}}{dt}, \qquad C_s = \frac{w\ h\ \varepsilon_0}{d_s}$$

then $$j_0 = \frac{\varepsilon_0}{d_s} V_{RF}\,\omega, \qquad V_{RF} = \frac{j_0 d_s}{\varepsilon_0 \omega}$$

Since $$d_s = \left(\frac{2V_{RF}\varepsilon_0}{q n_e}\right)^{1/2} = \frac{2 j_0}{\omega q n_e}$$ then

$$V_{RF} = j_0 \left(\frac{2V_{RF}\varepsilon_0}{q n_e}\right)^{1/2} \frac{1}{\varepsilon_0 \omega}, \quad V_{RF} = \frac{2 j_0^2}{\varepsilon_0 q \omega^2 n_e}$$

T_e The electron temperature comes from the ion balance:

$$n_e \cdot k_{ion} \cdot N_{gas} \cdot l_B \cdot w \cdot h = n_I \cdot u_B \cdot w \cdot h \cdot 2$$

$$k_{ion}(T_e) \cdot N_{gas} \cdot l_B = u_{Bohm} \cdot 2 = \left(\frac{kT_e}{M_I}\right)^{1/2} \cdot 2$$

n_e The electron density is obtained by equating all sources of electron heating to the power dissipation

(Power into e's) = ∑ (elastic + inelastic losses)

$$P_B + P_{sh} + P_R = n_e N_G \left(\frac{2m_e}{M}\,\frac{3}{2}\,k_{Bolt} k_{mom}(T_e - T_g) + \sum_i \Delta\varepsilon_i k_I\right) \cdot l_B \cdot w \cdot h$$

EXAMPLE: Argon RF discharge, 100 m Torr

$w = 2\pi \cdot 13.56$ MHz

$j_0 = 25\ \dfrac{\text{mA}}{\text{cm}^2}$, $L = 5$ cm, $\gamma = 0.05$

$M = 40$ AMU $R_{electrode} = 10$ cm

$$k_{ion} \approx 10^{-8}\ T_e^{1/2} \left(\frac{16\text{eV}}{T_e} + 1\right) \exp\left(\frac{-16}{T_e}\right) \frac{\text{cm}^3}{\text{s}}, \qquad \Delta\varepsilon = 16 \text{ eV}$$

$$k_{exc} \approx 2 \times 10^{-8}\ T_e^{1/2} \left(\frac{12\text{eV}}{T_e} + 1\right) \exp\left(\frac{-12}{T_e}\right) \frac{\text{cm}^3}{\text{s}}, \qquad \Delta\varepsilon = 12 \text{ eV}$$

$$k_{mom} \approx 5 \times 10^{-8}\ \frac{\text{cm}^3}{\text{s}}$$

T_e If we approximate $l_B = L - 2d_s \approx L$, then

$$k_{ion} \cdot N_{gas} \cdot l_B \approx u_{Bohm} \cdot 2 = \left(\frac{kT_e}{M_I}\right)^{1/2} \cdot 2 \approx k_{ion} \cdot N_{gas} \cdot L$$

$$3.2 \times 10^{16}\ \text{cm}^{-3}\ 10^{-8}\ T_{eV}^{1/2} \left(\frac{16\text{eV}}{T_{eV}} + 1\right) \exp\left(\frac{-16\text{eV}}{T_e}\right) L = \left(\frac{1.6 \times 10^{-12} T_{eV}}{M_{ion}}\right)^{1/2} \cdot 2$$

$T_e = 1.73$ eV

n_e $P_B + P_{stochastic} + P_e =$ elastic + inelastic

(Assume $\gamma = 0$ for n_e …)

$$hw \left[\frac{1}{2} j_0^2 \frac{m_e \nu_m}{q^2 n_e} L + \frac{m_e V_{th}}{q^2 n_e} j_0^2\right]$$
$$= \left[n_e\ N_G\ \nu_m \left(\frac{2m_e}{M}\right) \frac{3}{2} k_B (T_e - T_g) + n_e N_G (\Delta\varepsilon\ k_{ion} + \Delta\varepsilon\ k_{exc})\right] hwL$$

$n_e \approx 2.38 \times 10^{10}$ cm^{-3}

d_s (Sheath Width)

$$d_s = \frac{2j_0}{\omega q n_e} = \frac{2 \times 25 \times 10^{-3}}{2\pi \times 13.56 \times 10^6 \times 1.6 \times 10^{-19} \times 2.38 \times 10^{10}} \; \frac{\text{c} - \text{s cm}^3}{\text{cm}^2 - \text{s} - \text{c}}$$

$$= 0.154 \text{ cm} \quad (<< L = 5 \text{ cm})$$

(Note: With the known value of d_s, you can now replace L with $l_B = L - 2d_s$ in the expression for T_e to get a more accurate temperature.)

V_{RF}

$$V_{RF} = \frac{j_0 d_s}{\varepsilon_0 \omega} = \frac{25 \times 10^{-3} \;\; 0.154}{8.85 \times 10^{-14} \times 2\pi 13.56 \times 10^6} \; \frac{\text{c} - \text{cm J} - \text{cm} - \text{s}}{\text{cm}^2 - \text{s} - \text{c}^2}$$

$$= 511 \text{ V}$$

P_{total}

P_{bulk} =

$$\frac{1}{2} j_0^2 \left(\frac{m_e \nu_m}{q^2 n_e}\right)(L - 2d_s)\pi R^2 = \frac{(25 \times 10^{-3})^2 \, 0.911 \times 10^{-27} \;\; 1.6 \times 10^8 (5 - 2 \times 0.154)}{2(1.6 \times 10^{-19})^2 (2.38 \times 10^{10})} \; \frac{\text{ergs}}{\text{cm}^2 - \text{s}} \, \pi R^2$$

$$= 35.1 \; \frac{\text{mW}}{\text{cm}^2} \cdot \pi R^2 = 11.0 \text{ W} \qquad (7.4\%)$$

$$P_{\substack{\text{sheath} \\ \text{stochastic}}} = \frac{m_e \;\; V_{th}}{q^2 n_e} \, j_0^2 \, \pi R^2 = \frac{2 \times 0.911 \times 10^{-27} \;\; 1.12 \times 10^8 (25 \times 10^{-3})^2}{(1.6 \times 10^{-19})^2 \, 2.38 \times 10^{10}} \; \frac{\text{ergs}}{\text{cm}^2 - \text{s}}$$

$$= 20.9 \, \frac{\text{mW}}{\text{cm}^2} \cdot \pi R^2 = 6.58 \text{ W} \qquad (4.4\%)$$

$$P_{\substack{\text{ion} \\ \text{acceleration}}} = 2 \;\; n_e \;\; u_{Bohm} \left(\frac{qV_{RF}}{2} + \frac{kT_e}{2}\right) \cdot \pi R^2$$

$$= 2 \times 2.38 \times 10^{10} \left(\frac{1.6 \times 10^{-12} \;\; 1.73}{40 \times 1.67 \times 10^{-24}}\right)^{1/2} \left(\frac{1.6 \times 10^{-12} \;\; 511}{2} + \frac{1.6 \times 10^{-12} \;\; 1.73}{2}\right) \pi R^2$$

$$= 397 \; \frac{\text{mW}}{\text{cm}^2} \cdot \pi R^2 = 125 \text{ W} \qquad (84.0\%)$$

$$P_{sec} = 2 \cdot n_e \cdot u_{Bohm} \cdot \frac{qV_{RF}}{2} \cdot \gamma \pi R^2 = 6.2 \text{ W} \qquad (4.2\%)$$

$$P_{total} = 148.8 \text{ W}$$