Derivation of
Superelastic Cross Section

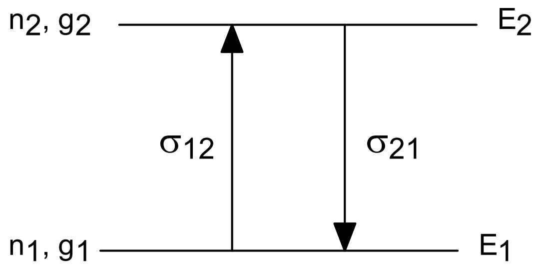
A "superelastic" collision (or collision of the second kind) occurs when an electron de-excites a higher lying state. In doing so, the electron gains energy

Inelastic	Superelastic
$e + N \rightarrow N^* + (e - \Delta\varepsilon)$	$e + N^* \rightarrow N + (e + \Delta\varepsilon)$

where $\Delta\varepsilon$ is the energy loss (inelastic) or energy gain (superelastic) resulting from the collision. The superelastic cross section is obtained from the excitation cross section through the Klein-Rosseland formula.

Assume we have two atomic levels

n_i = density of level
g_i = degeneracy
E_i = energy of level
$\Delta\varepsilon = E_2 - E_1$
σ_{12} = excitation cross section
σ_{21} = superelastic cross section



For a Maxwell Boltzmann electron energy distribution the number of electrons in $(\varepsilon, \varepsilon + d\varepsilon)$ is proportional to

$$f(\varepsilon)d\varepsilon = K\exp\left(\frac{-\varepsilon}{kT_e}\right)\varepsilon^{1/2}d\varepsilon$$

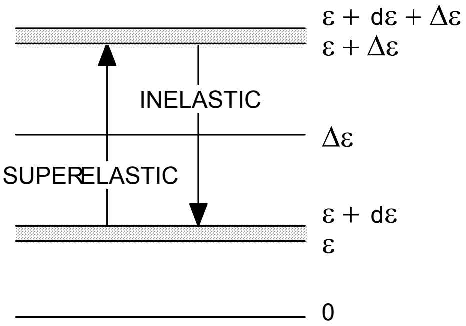
where K is a constant. The total rate of excitation of n_2 by these electrons is

$$R_{12}(\varepsilon)d\varepsilon = K\sigma_{12}(\varepsilon)n_1\left(\frac{2\varepsilon}{m_e}\right)^{1/2} f(\varepsilon)d\varepsilon \qquad \frac{\#}{cm^3 s}$$

The total rate of de-excitation is

$$R_{21}(\varepsilon)d\varepsilon = K\sigma_{21}(\varepsilon)n_2\left(\frac{2\varepsilon}{m_e}\right)^{1/2} f(\varepsilon)d\varepsilon \qquad \frac{\#}{cm^3 s}$$

If there is only one inelastic process, then electrons can only enter (ε, ε + dε) with ε < Δε by suffering inelastic collisions from higher energies, and can only leave (ε, ε + dε) by suffering superelastic collisions.



So in the steady state we must have the rate of population and depopulation of (ε, ε + dε) be equal.

$$\phi_{\mathrm{INTO}(\varepsilon,\varepsilon+d\varepsilon)} = R_{12}(\varepsilon+\Delta\varepsilon) \qquad \left[\begin{array}{l}\text{Electrons at } \varepsilon+\Delta\varepsilon \\ \text{exciting } n_1 \text{ and} \\ \text{losing energy.}\end{array}\right]$$

$$\phi_{\mathrm{OUT\,OF}(\varepsilon,\varepsilon+d\varepsilon)} = R_{21}(\varepsilon) \qquad \left[\begin{array}{l}\text{Electrons at } \varepsilon \text{ de exciting } n_2 \\ \text{and gaining energy.}\end{array}\right]$$

$$\sigma_{12}(\varepsilon+\Delta\varepsilon)n_1\left(\frac{2(\varepsilon+\Delta\varepsilon)}{m_e}\right)^{1/2}\exp\left(\frac{-(\varepsilon+\Delta\varepsilon)}{kT_e}\right)(\varepsilon+\Delta\varepsilon)^{1/2} =$$

$$\sigma_{21}(\varepsilon)n_2\left(\frac{2\varepsilon}{m_e}\right)^{1/2}\exp\left(\frac{-\varepsilon}{kT_e}\right)\varepsilon^{1/2}$$

$$\sigma_{21}(\varepsilon) = \sigma_{12}(\varepsilon+\Delta\varepsilon)\left(\frac{\varepsilon+\Delta\varepsilon}{\varepsilon}\right)\frac{n_1}{n_2}\exp\left(\frac{-\Delta\varepsilon}{kT_e}\right)$$

If we are in equilibrium then $\frac{n_2}{n_1} = \frac{g_2}{g_1}\exp\left(\frac{-\Delta\varepsilon}{kT_e}\right)$ so

$$\sigma_{21}(\varepsilon) = \frac{g_1}{g_2}\left(\frac{\varepsilon+\Delta\varepsilon}{\varepsilon}\right)\sigma_{12}(\varepsilon+\Delta\varepsilon)$$

$\sigma_{21}(\varepsilon)$ is always nonzero for ε > 0 since $\sigma_{12}(\varepsilon + \Delta\varepsilon)$ is nonzero.