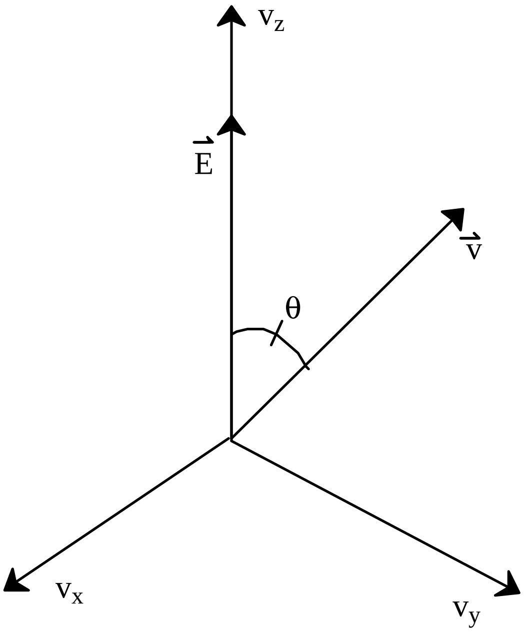
DRIFT VELOCITY FROM BOLTZMANN'S EQUATION



From the two-term spherical harmonic expansion

$$f(\vec{v}) = f_0(v) + f_1(v)\cos\theta$$

$$\int_0^\infty \int_0^\pi f_0(v) 2\pi v^2 \sin\theta \, d\theta \, dv = 1$$

$$f_1(v) = \frac{qE}{m_e \nu_m(v)} \frac{\partial f_0(v)}{\partial v}$$

Drift velocity for $\vec{E} = E\hat{z}$ is v_z.

$$v_d = \langle v_z \rangle = \langle v\cos\theta \rangle = \int_0^\infty \int_0^\pi \left(f_0(v) + f_1(v)\cos\theta\right) v\cos\theta \, 2\pi v^2 \sin\theta \, d\theta \, dv$$

Since f_0 is isotropic, $\int f_0 \cos\theta \, d\Omega$ integrates to zero.

$$v_d = \int_0^\infty \int_0^\pi \frac{qE}{m\nu_m} \frac{\partial f_0}{\partial v} 2\pi v^3 \cos^2\theta \sin\theta \, d\theta \, dv$$

$$v_d = \left[-\frac{4\pi}{3m_e} \int_0^\infty \frac{\partial f_0}{\partial v} \frac{1}{\nu_m(v)} v^3 dv \right] E = \mu E$$

where the mobility $\mu = \left[-\frac{4\pi}{3m_e} \int_0^\infty \frac{\partial f_0}{\partial v} \frac{1}{\nu_m(v)} v^3 dv \right]$

If $\nu_m(v)$ = constant, then

$$\mu = \frac{q}{m_e \nu_m} \left[\int_0^\infty -\frac{4}{3}\pi \frac{\partial f_0}{\partial v} v^3 dv \right] = \frac{q}{m_e \nu_m}$$

where the brackets integrate to 1.

DIFFUSION FLUX AND COEFFICIENT FROM BOLTZMANN'S EQUATION

From the two-term spherical harmonic expansion,

$$\frac{\partial f_1}{\partial t} + v\frac{\partial f_0}{\partial z} - \frac{qE_z}{m_e}\frac{\partial f_0}{\partial v} = -\nu_m f_1$$

where $\int f_0\, 2\pi v^2 \sin\theta\, d\theta = n_0$, the average electron density.

Set $\frac{\partial}{\partial t} = 0$ and E = 0 (only "thermal motion")

$$v\frac{\partial f_0}{\partial z} = -\nu_m f_1, \qquad f_1 = \frac{-v}{\nu_m}\frac{\partial f_0}{\partial z}$$

The diffusion flux in the z direction is $\Gamma_z = n_0 v_z = n_0 < v \cos\theta > = \int_0^\infty \int_0^\pi v_z (f_0 + f_1 \cos\theta)\, d^3v$.

Since $\int f_0 \cos\theta\, d\Omega$ integrates to zero, then

$$\begin{aligned}
\Gamma_z &= \int_0^\infty \int_0^\pi v_z f_1 \cos\theta d^3 v \\
&= \int_0^\infty \int_0^\pi v \cos\theta \left[\frac{-v}{\nu_m}\frac{\partial f_0}{\partial z}\right] \cos\theta\; 2\pi v^2 \sin\theta\, d\theta\, dv \\
&= -\int_0^\infty \int_0^\pi \frac{2\pi v^2}{\nu_m}\frac{\partial f_0}{\partial z} v^2 \cos^2 \sin\theta\, d\theta\, dv \\
&= -\int_0^\infty \frac{4\pi v^2}{3}\frac{v^2}{\nu_m}\frac{\partial f_0}{\partial z} dv = -n
\end{aligned}$$

$$\Gamma_z = -\frac{\partial}{\partial z}\left[\int_0^\infty \frac{4}{3}\pi v^2 \frac{v^2}{\nu_m} f_0 dv\right]$$

Now returning to the momentum conservation equation we have

$$\frac{\partial(nv)}{\partial t} = -\frac{\nabla P}{m} - nv\nu_m = 0$$

$$-\frac{\nabla nkT}{m} = nv\nu_m$$

$$\Gamma_z = nv_z = -\frac{\nabla nkT}{m\nu_m} = -\frac{kT}{m\nu_m}\nabla n = -D\nabla n$$

where we assume T = constant and the diffusion coefficient D = $\frac{kT}{m\nu_m}$ (also called the Einstein relation).

So we see that obtaining the conventional expression for diffusion flux as described by Ficks Law requires as a minimum that temperature T not be a function of position. However to go from our expression for Γ_z to Ficks Law we must also assume the $\nu_m(v)$ = constant and the $f_0(v)$ is a Maxwellian. We then have

$$\Gamma_z = -\frac{1}{\nu_m}\frac{\partial}{\partial z}\left[\int_0^\infty \frac{4}{3}\pi v^2 f_0 v^2 dv\right] = -\frac{1}{\nu_m}\frac{\partial}{\partial z}\left[\frac{nkT}{m}\right]$$

$$= -\frac{kT}{m\nu_m}\frac{\partial n}{\partial z} = -D\frac{\partial n}{\partial z}$$