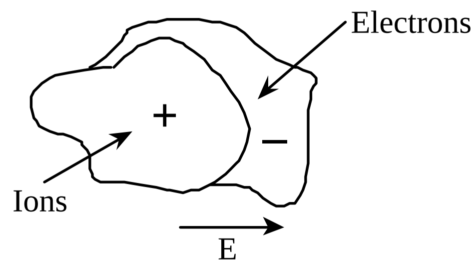
EECS 517/NERS 578 - HOMEWORK #1

A plasma stays neutral on a global basis because of the electrostatic forces which build up between negative and positive charges as they drift apart. Since the electrons are so much lighter than the ions, the electrons oscillate about the ions at the plasma frequency

$$\omega_p = \left(\frac{q^2 n_e}{m_e \varepsilon_0}\right)^{1/2} \left(\frac{\text{radians}}{\text{sec}}\right)$$



q = elementary charge
n_e = electron density
m_e = electron mass
ε_0 = permittivity of free space

The plasma frequency can be derived from many different approaches. Here we derive the plasma frequency by computing the acceleration resulting from "exposed" charge when the electrons oscillate away from the positive ions

$$\frac{\partial E}{\partial x} = \frac{\rho}{\varepsilon_0}, \quad \frac{\partial E}{\partial t} = \frac{\partial x}{\partial t}\frac{\partial E}{\partial x} = v_e \frac{\partial E}{\partial x} = v_e \frac{\rho}{\varepsilon_0}, \quad v_e = electron\ speed$$

$$\frac{\partial v_e}{\partial t} = \frac{-qE}{m_e}, \quad \frac{\partial^2 v_e}{\partial t^2} = \frac{-q}{m_e}\frac{\partial E}{\partial t} = \frac{-q}{m_e} v_e \frac{\rho}{\varepsilon_0} = \frac{-q^2 n_e}{m_e \varepsilon_0} v_e$$

$$v_e(t) = v_0 \sin(\omega_p t), \quad \omega_p = plasma\ \ frequency = \left(\frac{q^2 n_e}{m_e \varepsilon_0}\right)^{1/2} (MKS)$$

The distance over which the plasma is not neutral is the distance traveled by the electron in $\frac{1}{4}$ of its oscillatory cycle. This is the Debye Length.

$$\lambda_D = \left(\frac{\varepsilon_0 k_B T_e}{q^2 n_e}\right)^{1/2} (MKS)$$

$$\lambda_D \approx \left(\frac{2\pi}{\omega_p}\frac{1}{4}\right) v_e, \quad \frac{1}{2} m_e v_e^2 = k_B T_e \ (in\ 1D)$$

Where T_e is the electron temperature.

1) What is the peak magnitude of the electric field (V/cm) which constrains the plasma to being neutral? Parameterize and plot the Debye length and electric field as a function of electron density (10^8 cm^{-3} to 10^{15} cm^{-3}) and electron temperature (0.1 eV to 10 eV).

a. Plot λ_D (cm) vs. n_e (cm^{-3}) for different values of T_e (0.1, 0.3, 1, 3, 10 eV). You should use a log-log plot.
b. Plot the electric field E (V/cm) vs. n_e (cm^{-3}) for different values of T_e (0.1, 0.3, 1, 3, 10 eV). You should use a log-log plot.

2) A plasma is a conductor of electricity just as a wire or doped semiconductor is a conductor. For any conductor the current density for an applied field E is,

$$\vec{j}\left(\frac{A}{cm^2}\right) = \sigma\vec{E}, \quad \sigma = conductivity\left(\frac{1}{\Omega - cm}\right) = \frac{n_e q^2}{m_e \nu_m}$$

Where υ_m is the collision frequency which with electrons collide with atoms and molecules, and which randomizes their direction of motion. The current density is a vector quantity which points in the direction of the electric field. Since electrons have a negative charge, the direction of motion of electrons is opposite that of the current. In a plasma there is also a random electron current which results from the random thermal motion of the electrons just as molecules in air have a random thermal motion. This thermal current, j_{th}, has no net <u>vector</u> value, however its RMS value is

$$j_{th} = n_e q v_{th}$$

where v_{th} is the random thermal speed of the electrons.

a. What is the magnitude of j_{th} (A/cm^2) for electron densities of 10^8 cm^{-3} < n_e < 10^{15} cm^{-3} and for electron temperatures of 0.1 eV < T_e < 10 eV? Plot j_{th} (A/cm^2) vs n_e (cm^{-3}) for different values of T_e (0.1, 0.3, 1, 3, 10 eV). You should use a log-log plot.

b. What is the electric field you would need for the directed drift current density to be equal the random thermal current density? Plot the E field (V/cm) vs. n_e (cm^{-3}) for different values of T_e (0.1, 0.3, 1, 3, 10 eV). You should use a log-log plot.

In this problem, you can assume:

$$\frac{n_e}{N} = 10^{-5} \text{ (fractional ionization)}$$

$$\nu_m = 10^9 \, P_{Torr} \, s^{-1}$$

$$V_{th} = \left(\frac{8kT_e}{\pi m_e}\right)^{1/2} \frac{cm}{s}$$

where N = total gas density (cm^{-3})

ν_m = electron momentum transfer collision frequency

P = gas pressure (Torr)

k = Boltzmann's constant

m_e = electron mass

Is there a convenient scaling parameter for the electric field?