EECS 517 / NERS 578 FALL 2010 HOMEWORK #2

In this homework assignment, we investigate the angular deflection of an electron during a collision. We found that the angular distribution of the scattered electron during a collision can have measurable effect on its rate of momentum transfer and so the circuit-properties (e.g., current and voltage) of a gas discharge.

Assume:

$$f(\varepsilon)d\varepsilon = \frac{2}{\pi^{1/2}(k_B T_e)^{3/2}}\varepsilon^{1/2}\exp\left(-\frac{\varepsilon}{k_B T_e}\right)d\varepsilon$$

$$\sigma_T(\varepsilon) = \frac{10^{-15}}{\varepsilon_{\mathrm{eV}}}\ cm^2 = 2\pi\int_0^\pi \sigma_0(\varepsilon,\theta)\sin(\theta)d\theta$$

$$\sigma_0(\varepsilon,\theta) = \frac{A(\varepsilon)}{\varepsilon}\cos^n\left(\frac{\theta}{2}\right)$$

where

$\varepsilon \equiv$ electron energy

$f(\varepsilon) \equiv$ Electron energy distribution function. Here we assume that $f(\varepsilon)$ is a Maxwell-Boltzmann distribution.

$\sigma_T(\varepsilon) \equiv$ Total electron collision cross section. .(Note, $\sigma_T(\varepsilon = 1$ eV$) = 10^{-15}$ cm^2, $\sigma_T(\varepsilon = 10$ eV$) = 10^{-16}$ cm^2)

$\sigma_0(\varepsilon, \theta) \equiv$ Differential cross section. There is no explicit dependence of the differential cross section on the azimuthal (or ϕ angle). The ϕ dependence has been integrated out and that value is contained in the normalization constant.

$A(\varepsilon) \equiv$ Normalization constant

T_e = Electron temperature

k_B = Boltzmann's constant (1.38×10^{-16} erg/K, 1.6×10^{-12} erg/eV)

θ = Angular deflection in the polar direction of an electron following the collision (measured with respect to its velocity prior to the collision)

1. Plot $\sigma_0(\varepsilon, \theta)$ for $\varepsilon = 3$ eV as a function of θ from $(0,\pi)$ with $n = 0, 1, 3, 10$. Comment on the significance of n. Why do we plot the cross section only from $(0,\pi)$ and not $(0,2\pi)$?

2. Plot the relative number of electrons scattering into the solid angle centered on θ from $(0,\pi)$ with $n = 0, 1, 3, 10$, and $\varepsilon = 3$ eV. Comment on the results. Into what angle is the largest number of electrons scattered? Why?

3. Plot the electron momentum transfer collision frequency ν_m for a gas pressure of P = 5 Torr, gas temperature $T_g = 300$ K, and over the range of $0 < T_e < 10$ eV with $n = 0, 1, 3, 10$. Comment on the results and the significance of n.

HINTS: a. Solve for A(ε) from $\sigma_T(\varepsilon) = 2\pi \int_0^{\pi} \sigma_0(\varepsilon, \theta) \sin(\theta) d\theta$

b. Use $\frac{\nu_m}{N} = k_m = \int_0^{\infty} f(\varepsilon) \left(\frac{2\varepsilon}{m_\varepsilon} \right)^{1/2} \sigma_m(\varepsilon) d\varepsilon$ where the momentum transfer cross section is

$$\sigma_m(\varepsilon) = 2\pi \int_0^{\pi} \sigma_0(\varepsilon, \theta)(1 - \cos(\theta)) \sin(\theta) d\theta$$

4. If the current density, *j*, $\left(j = q n_e \mu_e E, \quad \mu_e = \frac{|q|}{m_e \nu_m} \right)$ and electron density, n_e, are constant what happens to the electric field as *n* increases from 0 to 10. Why?