

## EECS 517 / NERS 578 FALL 2010 HOMEWORK #3

The purpose of this assignment is to provide you with some insights for the magnitudes of electron impact rate coefficients as a function of electron temperature,  $T_e$ ; and how power is deposited in a glow discharge. You will find that at low  $T_e$  the electrons do not lose much energy per collision because they are not energetic enough to undergo inelastic collisions by exciting or ionizing the gas. As  $T_e$  increases and the rates of excitation and ionization increases, the energy loss-per-collision also increases.

Recall that a Maxwell Boltzmann electron energy distribution is given by:

$$f(\varepsilon) = \frac{2}{\pi^{1/2} (kT_e)^{3/2}} \varepsilon^{1/2} \exp(-\varepsilon/kT_e) \text{ eV}^{-1}$$

and that rate coefficients are obtained from:

$$k(T_e) = \int_0^\infty f(\varepsilon) \left( \frac{2\varepsilon}{m_e} \right)^{1/2} \sigma(\varepsilon) d\varepsilon \text{ cm}^3/\text{s}$$

1. Use the cross sections for the "ideal molecule" and plot the electron impact rate coefficients for the processes listed below as a function of electron temperature,  $T_e$ . Use a Maxwell-Boltzmann for the electron energy distribution function. Plot the rate coefficients,  $k(T_e)$ , for  $0.1 \text{ eV} \leq T_e \leq 10 \text{ eV}$ .

Comment on the results and discuss why  $k(T_e)$  has the shape it does for each process. (Hint: All rate coefficients can be obtained analytically. I encourage you to get the analytic solution. There is "physics" in the form of the analytic solution.

- a) Momentum transfer (use the elastic cross section)
- b) Vibrational excitation
- c) Ionization

### NOTES:

- a. Please display rate coefficients with units of  $\text{cm}^3/\text{s}$ .
- b. Use log-scales for the axes of your plots (as appropriate) to show the full dynamic ranges of variables and computed quantities. Also plot only "reasonable" ranges of the computed values. For example, a range of at most  $10^6$  from the maximum value is "reasonable"... a range of  $10^{30}$  is not reasonable. For example, if you have a rate coefficient of  $k = 10^{-30} \text{ cm}^3/\text{s}$  and a gas density of 0.5 Torr (about  $N = 10^{16} \text{ cm}^{-3}$ ), then the rate of collisions is  $v = kN = 10^{-14} \text{ s}^{-1}$  which is about once every 3 million years. This is a "zero" rate of collision!
- c. Please show your units analyses and intermediate steps with a "nice narrative" which explains your logic. )
- d. The "ideal molecule" cross sections were discussed in class and are available in the handout package from the website.

2. Define  $P_j(T_e)$  as the "power loss / collision" for process  $j$ . This is the average rate at which an individual electron loses energy due in collision process  $j$  with an individual atom.

$$P_j(T_e) = \int_0^\infty f(\varepsilon) \left( \frac{2\varepsilon}{m_e} \right)^{1/2} \sigma_j(\varepsilon) \Delta\varepsilon_j d\varepsilon \quad eV - cm^3 / s$$

where  $\Delta\varepsilon_j$  is the energy loss by the electron in collision  $j$ . The total power deposition in the plasma (per unit volume) resulting from electron collisions is then

$$P_T(T_e) = n_e N_g \sum_j P_j(T_e) = n_e N_g \sum_j \int_0^\infty f(\varepsilon) \left( \frac{2\varepsilon}{m_e} \right)^{1/2} \sigma_j(\varepsilon) \Delta\varepsilon_j d\varepsilon \quad W / cm^3$$

where  $n_e$  is the electron density and  $N_g$  is the gas density. The sum is over all the collisional processes (which in this case are momentum transfer, vibrational excitation and ionization.) Plot  $P_j(T_e)$  as a function of  $T_e$  for each individual collisional process (momentum transfer, vibrational excitation and ionization) for range of  $T_e$  as in Problem 1. Assume the gas has molecular weight  $M = 28$  AMU (as does  $N_2$ ) and a gas temperature of 300 K. Please comment on the results. Discuss the physics that is responsible for the trends for each process and discuss how power deposition is divided among the three processes as a function of  $T_e$ . (NOTE: The energy loss in an elastic collision with a gas atom having mass

$M$  by an electron with energy  $\varepsilon$  is  $\left( \frac{2m_e}{M} \cdot \varepsilon \right)$ . You can assume that the energy loss per collision for inelastic processes is the threshold energy.)

3. Plot the total power deposition  $P_T(T_e)$  [ $W/cm^3$ ] for the conditions of Problem 2 for a gas pressure of  $P = 5$  Torr,  $T_g = 300$  K and  $n_e = 10^{10} cm^{-3}$ . Please comment on the results. A typical fluorescent lamp (a gas discharge lighting source, 1.5 inches in diameter and 4 feet long) operates at about 7 Torr and dissipates about 40 W of power. Estimate the electron density assuming the plasma is sustained in the ideal molecule.

4. Assume that vibrational excitation of the ideal molecule results in an attachment (that is, the loss of an electron by forming a negative ion). In the steady state (ignoring other electron sources and losses) we must have  $\frac{\partial n_e}{\partial t} = 0$ . What is the steady state electron temperature for a plasma sustained in a gas of ideal molecules for these conditions (that is, for the 3 processes of elastic collisions, vibrational excitation that results in an attachment and ionization)? Why?

**HINT:** DO NOT DO ANY ADDITIONAL CALCULATIONS FOR PROBLEM 4. YOU ALREADY HAVE THE ANSWER!