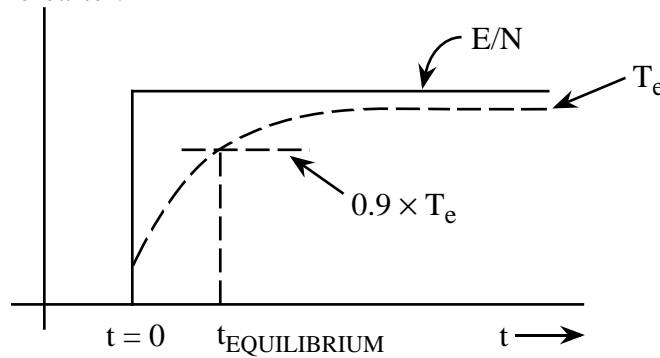


EECS 517 / NERS 578 FALL 2010 HOMEWORK #4

“Non-equilibrium” electron transport is important in low pressure electric discharge devices. “Equilibrium” transport usually refers to conditions where the local field approximation (LFA) is valid. The LFA states that, for a given gas mixture, the local instantaneous transport coefficients (e.g., electron temperature, ionization rate coefficients) can be uniquely specified by the value of the local instantaneous electric field (or E/N). This requires that the plasma is in the quasi-steady state [$dn/dt \approx 0$] and that spatial derivatives are not important [$dn/dx \approx 0$]. By quasi-steady state, we mean that the time that, for example, the electron temperature comes into equilibrium with changes in the electric field is short compared to the time over which the electric field changes. Therefore, the electron temperature “tracks” changes in E/N . In “non-equilibrium” transport, the electron temperature, T_e , does not directly track changes in E/N . That is, the equilibration time for T_e is longer than the time over which E/N changes. In this assignment we will investigate how long and over what distance an electron swarm comes into equilibrium with a change in the electric field. By doing so, we will be able to assess when the LFA is a good approximation.

1. Assume that at $t = 0$ the electric field makes a step function change from $0 \rightarrow (E/N)_0$ and remains constant thereafter.



By integrating the electron energy conservation equation as a function of time, calculate the time that T_e takes to come into equilibrium with the new value of E/N . You may assume that the “equilibration time” is the time when T_e reaches 90% of its asymptotic steady state value. The conditions you should use for your calculation are:

- $T_e(t = 0) = 0.05 \text{ eV}$,
- Mass of gas atom = 28 AMU
- $T(\text{gas}) = 300 \text{ K}$

- Use the Ideal Molecule cross sections with momentum transfer, electronic excitation and ionization processes.
- Assume the electron density remains constant.
- Recall that the units of $E/N = [\text{Electric field (V/cm)}] / [\text{Gas Density (1/cm}^3)] = \text{V-cm}^2$.
 E/N is often expressed in units of Townsend (Td) where $1 \text{ Td} = 10^{-17} \text{ V-cm}^2$.

Perform your calculations for

- $(E/N)_0 = 1, 10, \text{ and } 100 \text{ Td}$ at a gas pressure of 1 Torr.
- Gas pressure = 0.1, 1 and 10 Torr for $(E/N)_0 = 10 \text{ Td}$.

Please comment on your results and on the scaling of the equilibration time with $(E/N)_0$ and gas pressure.

[**HINT:** You need to integrate the expression $\frac{\partial T_e}{\partial t} = \text{function}(T_e)$, a first order ordinary-differential-equation (ODE), as a function of time. This is an initial value problem where you start with a known value of T_e and integrate (or march) forward in time, recording the value of T_e along the way. For example, a very simple integration scheme is

$T_e(t + \Delta t) = T_e(t) + \frac{\partial T_e(t)}{\partial t} \Delta t$. You will need to keep Δt small enough so that the fractional change in T_e in any timestep is small. You can use ODE solvers in applications such as MATLAB as well. The integration in this assignment *cannot* be done analytically or by an integration by parts. It must be done numerically.]

- Is there any way to "collapse" the results from Problem 1 into a more convenient form using a scaling parameter or scaling law?
- Over what distance does the electron swarm in Problem 1 move in the time required for T_e to come into equilibrium for:
 - $E/N = 1 \text{ Td}$, gas pressure = 0.1 Torr
 - $E/N = 100 \text{ Td}$, gas pressure = 100 Torr