

EECS 517 / NERS 578 FALL 2010 HOMEWORK #6

Ions drifting in electric fields obey the same relationships as do electrons. For example, the drift velocity, v_d , of ions in an electric field E is

$$v_d = \mu_I E$$

where the ion mobility is

$$\mu_I = \frac{q}{M k_m N},$$

k_m = momentum transfer rate coefficient for ions.

M = Mass of the ion

N = density of atoms (or molecules) with which the ion collides.

For ions we often define an effective temperature, T_{eff} ,

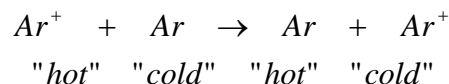
$$\frac{3}{2} k_B T_{eff} = \frac{3}{2} k_B T_I + \frac{1}{2} M v_d^2$$

where k_B is Boltzmann's constant and T_I is the Maxwellian ion temperature that accounts for random thermal motion. T_{eff} additionally accounts for the kinetic energy of the drifting ions. Using T_{eff} , we have,

$$k_m = \left(\frac{8 k_B T_{eff}}{\pi M_R} \right)^{1/2} \sigma_m \quad \text{cm}^3 \text{s}^{-1}$$

where σ_m is momentum transfer cross section (cm^2), and M_R is the reduced mass of the ion and its neutral collision partner. $M_R = \frac{M_1 M_2}{M_1 + M_2}$

1. Given the experimental data contained in Table I, compute T_{eff} and σ_m for Ar^+ drifting in Ar, and for Hg^+ drifting in He for the ranges of E/N listed. Plot and comment on the results. Why are the cross sections for Ar^+ and Hg^+ so different?
2. A swarm of Ar^+ ions are drifting in a weak electric field with a drift speed $v_d = 1 \times 10^4$ cm/s. The ions enter a sheath of width $L = 0.1$ cm adjacent to a quartz surface. The sheath potential is $\Phi = -30$ V. The mean free path between collisions of Ar^+ with Ar gas atoms (pressure = 100 mTorr) is $\lambda = 400$ μm . When the Ar^+ ion has a collision with an Ar gas atom it undergoes a symmetric charge exchange collision.



In this process, the ion enters into the collision being "hot" (having been accelerated in the electric field) and "exchanges" its charge with a "cold" gas atom. After the collision, the ion has been neutralized by producing a "hot" gas atom, whereas the once "cold" gas atom is now a "cold" ion. As a result of the collision, the ion appears to be "thermalized". That is, the ion loses its directed energy and is reduced to being a thermal ion with a random directed velocity. Another way of thinking of this is that during the collision, T_{eff} becomes T_g .

Estimate the distribution of ion energies striking the quartz surface. You can assume that $T_{gas} = 300$ K and $T_{ion} = 300$ K outside the sheath. Is this distribution of ion energies what you would expect from the effective temperature discussed in Problem 1? (For additional insights, you might want to repeat the calculation for $\lambda = 40$ μm and $\lambda = 4000$ μm .)

HINT: You can compute the ion energy distribution by integrating a large number of ion trajectories through the sheath and allowing for there to be charge exchange collisions at different locations in the sheath. For example, start an ion at the edge of the sheath and integrate its equation of motion.

$$\frac{dx}{dt} = v, \quad \frac{dv}{dt} = \frac{qE}{M}$$

"Randomly" choose the distance to the ion's next collision by, for example, having the distance vary by some uncertainty $d = \lambda \pm \Delta\lambda$. [If you choose a Poisson distribution for the collision mean free path, the distance to the next collision is $d = -\lambda \ln(r)$, where r is a random number between 0 and 1.] When the collision occurs, change the velocity to a thermal value, and continue with the integration until the ion hits the surface. By repeating this process for many (many) ions, you can build up an ion energy distribution. This is called a "Monte Carlo Simulation." An outline of a short program that will perform this calculation is shown below. If you were to write a short program to do this, it might look like the following. (Note: This is a "Monte Carlo Simulation".)

Note: Most compilers or programs such as MATLAB have a random number generator that gives you a different random number between (0,1) every time it is called.

Table I

Ar ⁺ drifting in Ar		
Experimental Data T = 300° K		
E/N	μ_0	v_d
8.00	1.53	0.329
10.0	1.53	0.411
12.0	1.53	0.493
15.0	1.52	0.613
20.0	1.51	0.811
25.0	1.49	1.00
30.0	1.47	1.18
40.0	1.44	1.55
50.0	1.41	1.89
60.0	1.38	2.22
80.0	1.32	2.84
100	1.27	3.41
120	1.22	3.93
150	1.16	4.68
200	1.06	5.70
250	0.99	6.65
300	0.95	7.66
400	0.85	9.14
500	0.78	10.5
600	0.72	11.6
800	0.63	13.5
1000	0.56	15.0
1200	0.51	16.4
1500	0.46	18.5
2000	0.40	21.5

Hg ⁺ drifting in He		
Experimental Data T = 300° K		
E/N	μ_0	v_d
4.00	19.7	2.12
5.00	19.7	2.65
6.00	19.6	3.16
8.00	19.6	4.21
10.0	19.5	5.24
12.0	19.5	6.29
15.0	19.4	7.82
20.0	19.2	10.3
25.0	19.2	12.9
30.0	19.2	15.5
35.0	19.3	18.2

E/N The ratio of electric field strength to the neutral-gas number density. The units are Townsend (Td), 1 Td = 10^{-17} V-cm².

v_d The ion drift velocity in the neutral gas in units of 10^4 cm/sec.

μ_0 The reduced mobility of the ion in the neutral gas, in units of cm²/V-sec. The reduced mobility is the value of the mobility at STP (1 atm pressure and $T_g = 0$ C = 273 K). To obtain the mobility μ at gas pressure P (Torr) and neutral gas temperature T_g (K) from the reduced mobility,

$$\mu(P, T_g) = \mu_0 \left(\frac{760}{P} \right) \left(\frac{T_g}{273} \right)$$

Outline of a program that will perform the Monte Carlo Simulation that computes an ion energy distribution striking the wall.

```

Tion=300.
Tgas=300.
xmass=40.*1.67e-24
xkb=1.38e-16
sheath_width=0.1
q=1.6e-19
c
c xlambda = mean free path between collisions. This can be changed in
c different cases.
c
c xlambda=400.e-4
c
c do 500 i = 1, total_number_of_ions
c   x = 0.
c   v = 1.e+4
c   vthermal=sqrt(8.*xkb*Tgas/(pi*xmass))
c
c   r = random number between (0,1) which is different each time
c   xlocation_of_next_collision is the randomly chosen location of the
c   next collision
c
c   xlocation_of_next_collision = x - xlambda*ln(r)
c   step_size=sheath_width/1000.
c
c   Perform integration of the ion trajectory
c
100   dt = step_size/v
c
c   E is the electric field based on the sheath potential and sheath width
c
c   acceleration = qE/xmass
c   x = x + v*dt + 0.5*acceleration*dt*dt
c   v = v + acceleration*dt
c   if (x.ge.sheath_width) then
c
c     Ion hit the wall. Record the energy of the ion in a histogram and
c     start a new ion at the
c     sheath edge.
c
c     energy=0.5*xmass*v**2
c     go to 500
c     end if
c     if (x.ge.xlocation_of_next_collision) then
c
c       Ion has had a charge exchange collision. Change its temperature to
c       that of the gas.with a direction either towards or away from the wall.
c
c       v = vthermal
c       random_number = r
c       if (random_number.le.0.5) v = -v
c       location_of_next_collision = x - xlambda*ln(r)
c     end if
c     go to 100
500 continue

```