Probability distributions for the breakdown voltage between closely spaced electrodes on insulating surfaces[a]

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Probability distributions for the breakdown voltage between closely spaced electrodes (≈ 10 mils) on insulating surfaces are studied with results from a Monte-Carlo simulation. The probability distributions, experimentally measured to be bimodal under certain conditions, are found to be characterized by the number of ionizations per primary electron emitted at the triple junction that is required to initiate the electron avalanche. Bimodal distributions represent a transition region between low variance and high variance normal distributions requiring low and high multiplication coefficients, respectively, for the avalanche to occur. Conditions of moderate preionization and low electron loss rates to the insulating surface (e.g., large secondary electron coefficient) are found to lower the breakdown voltage and reduce bimodal distributions to single normal distributions.

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I. INTRODUCTION

The breakdown between closely spaced electrodes (2–10 mil) on insulating surfaces in air has recently been investigated.[1] The motivation for that study is that high voltage testing is used in industry to detect conductor spacings of less than 5 mil on printed wiring boards. The voltage at which breakdown occurs between the narrow gaps is a function of the gap spacing, and therefore can be used as a quality control technique. The breakdown voltage has usually been assumed to be Gaussian (normally) distributed. It was found, though, that the distribution of breakdown voltages is often nonGaussian, and sometimes bimodal in nature.[1,2] The specifics of the distribution including the average breakdown voltage, are functions of the particular insulator on which the electrodes are mounted, the degree of preionization, and the number of times the gap has been previously broken down. From the standpoint of reliability, it would be desirable to be able to predict with certainty the breakdown voltage of a printed wiring board with a specified geometry. The variability in the breakdown probability distributions makes this difficult. In order to study the probability distribution for breakdown between closely spaced electrodes on insulating surfaces, a Monte-Carlo simulation computer code has been written and exercised. The results of the study will be discussed below.

The breakdown between electrodes on an insulating surface fundamentally differs from the breakdown between identical electrodes in the absence of the surface. At the metal-insulator-gas junction (the "triple junction"), large electric field concentrations can occur.[3] The field values are sufficiently high that primary electrons can be emitted from the junction. Once the primary electron is emitted into the gas, the properties of the insulating surface become critical. An electron scattered into or attracted to the surface can be collected, reflected, or can cause the emission of secondary electrons. Therefore, depending on the characteristics of that surface, it can represent either a source or a sink of electrons.

During the prebreakdown stage, the entire applied voltage appears across the electrodes. The drift velocity of electrons after being emitted from the triple junction can, therefore, be large even when the surrounding gas is at atmospheric pressure. As a result, an electron can have a large probability of traversing the narrow gap and being collected by the anode without experiencing many ionizing collisions. These electrons, as well as electrons which are scattered out of the volume between the electrodes or collected by the insulating surface, can be thought of as having been lost to a sink. We can define an effective multiplication coefficient γ for the electrons emitted from the triple junction:

$$\gamma = 1 + \int_0^l [(\alpha_1 + \alpha_2) - (\beta_1 + \beta_2)]\, dx. \qquad (1)$$

In (1), α_1 is the ionization coefficient for the production of electrons by collisions with the gas, α_2 the effective ionization coefficient for the production of electrons by secondary processes, β_1 the coefficient for the loss of electrons by attaching to the gas, β_2 the coefficient for the loss of electrons by collection by surfaces or scattering out of the volume, and l the gap spacing. Clearly the criteria for an electron avalanche to occur between the electrodes, which will initiate breakdown, is that $\gamma > 1$. Of the parameters in (1), α_1 and β_1 are properties of the gas, while α_2 and β_2 are properties of the surfaces and geometry. We will see that the value of γ which is necessary to initiate the electron avalanche characterizes the probability distribution for breakdown.

The experimental results of Ref. 1 will be briefly discussed as an introduction to the Monte-Carlo simulations. In that study, breakdown voltages were measured using copper electrodes mounted on epoxy-glass and triazine surfaces. The gap spacing was 2–10 mils in air at 0.2 to 1.0 atmosphere pressure. A ramping voltage (200–400 V/s) was applied to the electrodes and the voltage at breakdown recorded. Three

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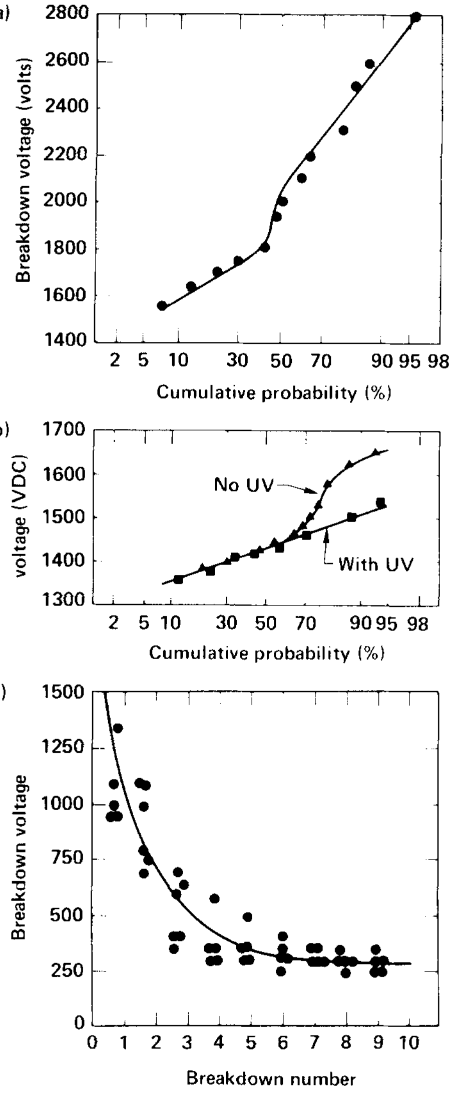

FIG. 1. Typical experimental results from Ref. 1. (a) Example of bimodal probability distribution function. (b) Reduction of bimodal probability distribution function to a single normal distribution by UV preionization. (c) Decrease in average breakdown voltage as the number of previous breakdown events increases.

issues from that study will be addressed here. First, the probability distributions for breakdown were often found to be bimodal. This is illustrated in Fig. 1(a). (A normal distribution is a straight line when plotted on the scales used for these plots.) The bimodal distribution is represented by the two straight line segments which define separate mean values and variances. Secondly, preionization of the gas was found to not only lower the average breakdown voltage of the gas under certain conditions, but also to reduce a bimodal distribution to a single distribution [Fig. 1(b)]. Thirdly, the average breakdown voltage was found to decrease as the number of times a gap had been previously broken down increased [Fig. 1(c)]. This phenomenon occurred with some surfaces (eg., epoxy glass), but not others (eg., mica-phenolic).

II. THE MONTE-CARLO SIMULATION

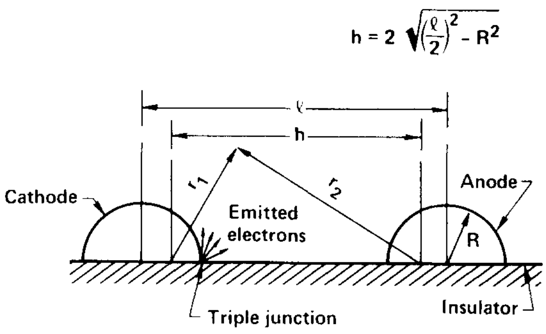

FIG. 2. Geometry for the Monte-Carlo simulation. Electrons are emitted from the triple junction with thermal energy and random angle ($0 < \theta < \pi/2$).

The geometry for the Monte-Carlo simulation is shown in Fig. 2. Cylindrical electrodes 4 mils in diameter on an insulating surface were placed with their centers 14 mils apart. The potential distribution between the electrodes was assumed to be independent of the surface properties and is given by[4]

$$V(r_1,r_2) = V_0 \ln(r_2/r_1)/2 \ln\left[\frac{l}{2R} + \left(\left(\frac{l}{2R}\right)^2 - 1\right)^{1/2}\right], \quad (2)$$

where V_0 is the applied voltage, R the radius of the electrode, and r_2 and r_1 defined in Fig. 2. A single Monte-Carlo simulation run consisted of the following procedure: an initial charging voltage was chosen and a group of electrons (typically about 400) were launched from the triple junction with thermal energy and random direction ($0 < \theta < \pi/2$). The progress of each electron was followed until it was collected by a surface or scattered out of the volume. This volume was defined by a hemisphere with a radius equal to five electrode spacings centered on the midpoint between electrodes. The number of ionizing collisions or secondary events was recorded, and the γ parameter calculated. If the parameter γ exceeded an arbitrarily selected critical value γ_0 (greater than one) before all the electrons were collected, then an electron avalanche and breakdown were assumed to have occurred. If all the electrons were collected before γ exceeded γ_0, then the charging voltage was increased and a new

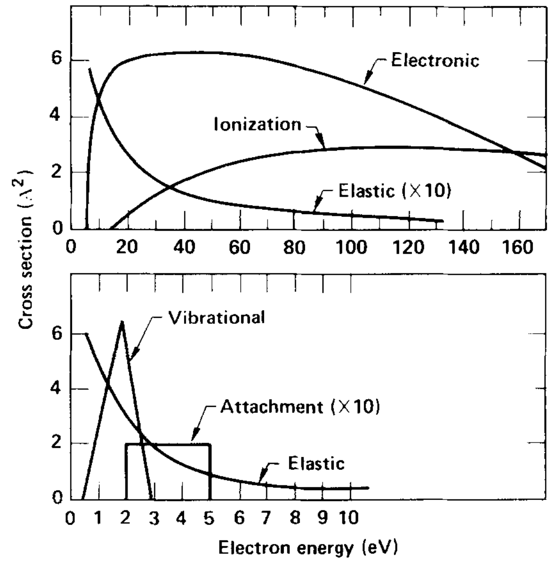

FIG. 3. Cross sections used in the Monte-Carlo simulation.

group of electrons was launched. This procedure continues until breakdown occurs. The incremental increase in voltage was either 33 or 50 V. A distribution of breakdown voltages was obtained from at least 50 separate Monte-Carlo runs.

The specific values for breakdown voltage obtained with the simulation, are, of course, functions of the particular cross sections used. The systematic behavior for breakdown under our conditions, though, was found to be relativiely insensitive to the details of the form of the cross sections. Therefore, for the purposes of this study, an ideal gas molecule was used. The inelastic thresholds for electron impact are at 0.5 eV (vibrational), 2.0 eV (attachment), 6.0 eV (electronic), and 14.0 (ionization) (see Fig. 3).

IV. BIMODAL PROBABILITY DISTRIBUTIONS FOR BREAKDOWN VOLTAGE

The critical multiplication coefficient γ_0 required for breakdown was found to characterize the probability distribution for breakdown voltages. Recall that γ is a function of the gas (α_1 and β_1), as well as the geometry and material properties (α_2 and β_2). A particular gas will, therefore, not have a unique γ. The distribution of breakdown voltages for the geometry of Fig. 2 for three different values of γ_0 is shown in Fig. 4. For these cases, the insulating surface was treated as an absorbing plane for electrons. As γ_0 increases, the average breakdown voltage and the variance of the distribution increases. For small γ_0, the distribution of breakdown voltages is described well by a normal distribution. At an intermediate γ_0, the distribution shows a definite bimodal character. At large γ_0, the distribution returns to being normal. Note that the lower part of the $\gamma_0 = 1.25$ distribution has about the same variance (i.e., is parallel to) the low γ_0 case, while the upper part of the $\gamma_0 = 1.25$ distribution has nearly the same variance as the high γ_0 case. It appears that bimodal distributions of breakdown voltages coincide with a transition between a normal distribution with a small variance to a normal distribution with a large variance. That is, the bimodal distribution represents a transition between conditions, where the number of ionization events per emitted electron required for breakdown is low ($\lesssim 1.1$) to conditions where the number of ionizations per emitted electron required for breakdown is high ($\gtrsim 1.5$).

The transition to large variance is initiated by an increase in γ_0, and occurs first at voltages greater than the mean. This effect can be qualitatively explained by referring to Fig. 5. For the results in this figure, groups of 400 electrons were launched from the triple junction for applied voltages between 1000 and 3000 V. The electrons were followed until they were all collected and the number of ionizations per electron emitted was recorded. As one would expect, the number of ionizations per electron increases as the voltage increases, but so does the statistical scatter of this value. The scatter in values is due in large part to a decrease in residence time of the electrons between the electrodes caused by an increase in the drift velocity as the applied voltage increases. Although the ionization rate constant increases with increasing applied voltage, the decrease in residence time prevents the number of ionizations per emitted electron from increasing by the same fraction. The increase in the directed velocity is reflected by the increase in the fraction of electrons collected by the anode. This fraction increases from 15% to 30% as the applied voltage increases from 1000 to 3000 V. Therefore, the rate of increase in the probability of breakdown decreases after a given voltage is surpassed. The increase in variance is due to the decrease in sampling time for each electron. As γ_0 increases, the required number of ionizations per electron emitted increases, thereby driving the required voltage into a region corresponding to small residence time and, therefore, higher variance of the resulting distribution.

Under our conditions, about 60–70% of the electrons emitted from the triple junction are scattered into the surface and collected, with this fraction decreasing slightly with increasing applied voltage. The fraction is large compared to the fraction of electrons collected by the anode as a result of the seed electrons being launched from the plane of the insulating surface. The majority of the electrons collected by the

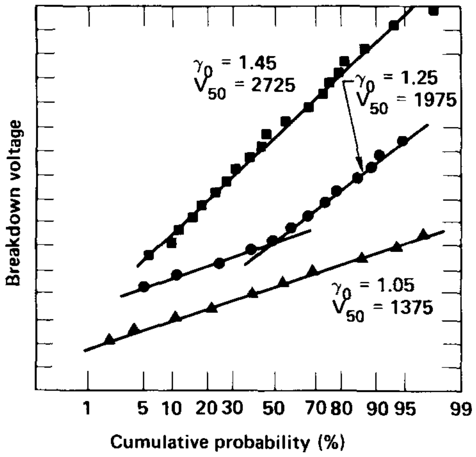

FIG. 4. Computed probability distribution functions of breakdown voltage for different values of γ_0, the multiplication coefficient required to initiate a breakdown avalanche. V_{50} is the breakdown voltage at 0.5 probability. The ordinate scale is in increments of 100 V.

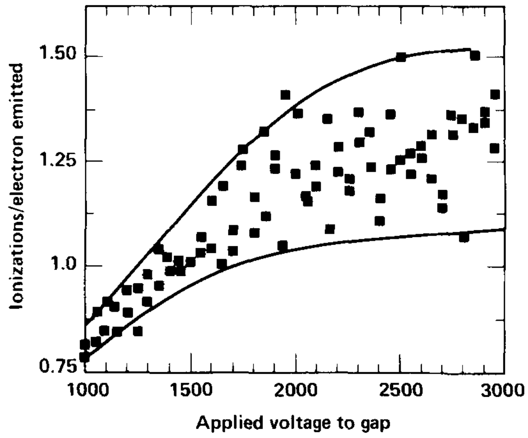

FIG. 5. The average number of ionizations per electron emitted from the triple junction as a function of applied voltage. (Each point is the average of 400 electrons). The increase in scatter as the voltage increases can be attributed to a reduced transit time for collection of electrons by the anode.

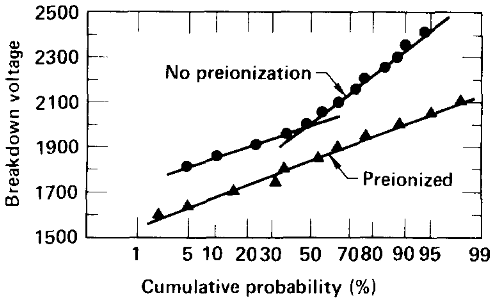

FIG. 6. Probability distribution function for breakdown voltage with and without preionization.

surface are lost after only a few randomizing collisions. As a result, preionization and surface properties can significantly effect the probability distribution for breakdown.

The computed probability distribution function for breakdown with and without preionization is shown in Fig. 6. The preionized case was simulated by randomly distributing 15% of the seed electrons in the gas between electrodes. As a result of preionization, the initially bimodal distribution is reduced to a single normal distribution, in agreement with the experimental results discussed above [Fig. 1(c)]. In the presence of preionization, the fraction of electrons lost to the insulating surface is decreased, and the fraction collected by the anode is increased. This effectively increases the residence time of electrons between the electrodes, thereby preventing a transition of the distribution into a region of high variance and bimodality.

IV. BREAKDOWN VOLTAGE AS A FUNCTION OF PREVIOUS BREAKDOWN EVENTS

The observation that the breakdown voltage of a closely spaced gap on an insulating surface decreases as the number of times the gap has broken down increases must be explained by some incremental change in the material properties of the surfaces or by a nondestructive but cumulative effect of breakdown. Clearly, any mechanism which increases the loss rate of electrons to the surface (eg., by decreasing the secondary emission coefficient) will increase the average breakdown voltage. Therefore, if the mechanism responsible for the effect described above is a function of the properties of the insulating surface, it must decrease the loss rate. We will discuss a nondestructive (passive) mechanism and a mechanism where the material properties of the surface are changed as explanations for the observed decrease in breakdown voltage.

A. Surface potential

A passive mechanism which qualitatively explains the experimental results is the successive accumulation of surface charge on the insulator. After breakdown has occurred and the discharge pulse is over, the dielectric insulating surface my retain a significant amount of negative charge. [It is well known that dielectric surfaces will charge up to a few times (negative) the average electron temperature when immersed in a plasma.] If this is the case, the surface will represent a reflecting surface to electrons with energy less than the (negative) potential at the beginning of the next breakdown trial. This reflection coefficient reduces the loss rate of electrons to the surface. The accumulation of surface charge during successive breakdown events and discharge pulses increases the electron reflection coefficient, and can therefore reduce the breakdown voltage of the next trial. To test this hypothesis with the Monte-Carlo simulation, a surface potential, $-V_s$, was specified for the insulator. Electrons with energy less than eV_s were reflected from the surface, while electrons with energy greater than eV_s were collected. The results of the simulation as a function of V_s are shown in Fig. 7. The systematic decrease in breakdown voltage with increasing V_s reproduces the experimentally observed decrease in breakdown voltage with increasing number of breakdown events. In addition to the reduction in breakdown voltage, the probability distribution, bimodal at low V_s, reduces to a single normal distribution with a small variance (50–60 V) as V_s increases. This observation is consistent with a decreasingly small β_2 (loss rate of electrons by nonattaching events), and an increase in residence time of electrons between the electrodes.

B. Secondary electron emission

The secondary emission coefficient for electrons by electron impact is a function of the incident electron energy and composition of the material. Typically the average number of electrons emitted per primary electron, δ, has a maximum value of 2–5 for an electron energy of a few hundred volts to a few thousand volts.[3,5] If on the average, δ is less than 1, then the surface appears to be a sink for electrons. If on the average $\delta > 1$, then the surface appears to be a source of electrons. The breakdown voltage therefore is a function of δ. If the δ of the insulating surface changes as a result of cummulative discharge induced damage, then the breakdown voltage will be a function of the number of previous

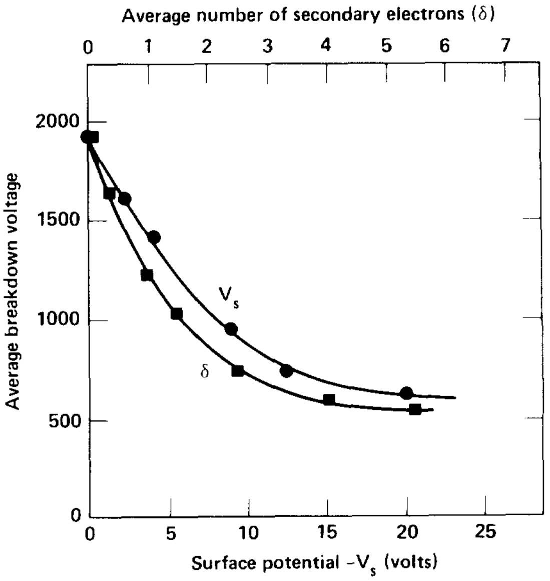

FIG. 7. Average breakdown voltage: (●) As a function of the (negative) potential of the insulating surface. (■) As a function of δ, the average number of secondary electrons emitted per energetically allowed primary electron collected by the insulating surface.

times the gap has been broken down. This mechanism was investigated with the model.

A value was assigned to δ in the Monte-Carlo simulation. This value was the average number of secondary electrons emitted by the surface per incident electron collected by the surface with energy greater than the work function (3.5 eV). Electrons with energy less than the work function were collected without secondary emission In the simulation, the integer number of secondary electrons emitted by the surface when a primary electron is collected is given by

$$n = \mathrm{Int}[-\ln(r)^*\delta], \tag{3}$$

provided that the electron energy is greater than n times the work function. In (3), r is a random number between 0 and 1, and Int is the integer function. This form for n is a conservative estimate for the secondary emission rate since $\langle n \rangle \leqslant \delta < \langle n \rangle + 1$.

Results from the Monte-Carlo simulation for the average breakdown voltage as a function of δ are also shown in Fig. 7. The average breakdown voltage decreases with increasing δ, reproducing the experimentally observed decrease in breakdown voltage with increasing number of breakdown events. Bimodal distributions are obtained with small δ, and single normal distributions with large δ, consistent with a decrease in β_2, and an increase in the average residence time of electrons in the gap.

V. CONCLUDING REMARKS

A Monte-Carlo simulation has been used to investigate the probability distribution function for breakdown between closely spaced electrodes on an insulating surface. Bimodal probability distributions for the breakdown voltage were found to correspond to a transition between a region where the number of ionizations per emitted electron required to initiate the breakdown cascade is small ($\lesssim 1.1$) to a region where this number is large ($\gtrsim 1.5$). The form of the distribution, as well as the average breakdown voltage, is a function of the state of preionization and the characteristics of the surface. Preionization of the gas will reduce a bimodal distribution to a single distribution, as will reduction of the electron loss rate to the insulating surface during the prebreakdown stage. Mechanisms which reduce the loss rate of electrons to the surface, such as the accumulation of surface charge or an incremental increase in the secondary emission coefficient, qualitatively explain the experimentally observed reduction in breakdown voltage as the number of breakdown events for a given gap increase. The value of δ (4–5) required to reduce the average breakdown voltage by the amount observed experimentally is within the range of values of δ for common materials. It is doubtful, though, that the value of the surface potential (-20 V) required to reduce the average breakdown voltage by the same amount can be easily obtained under the conditions of interest.

ACKNOWLEDGMENTS

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