Townsend coefficients for electron scattering over dielectric surfaces

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A method for describing the probability of initiating flashover discharges across dielectric surfaces is presented in which we define a transport coefficient for electron multiplication similar to the Townsend coefficient used for gas discharges. The coefficient is a function of the scaling parameter (charge released from the cathode)/(cathode-anode separation) and is also a measure of the growth of the sheath on the dielectric surface resulting from electron scattering. We discuss results for when the source of seed electrons does not necessarily depend upon field emission at the cathode-vacuum-dielectric triple point. This may occur when the surface is illuminated by UV radiation. For these conditions, there is a different functional dependence of flashover probability on voltage and geometry (e.g., thickness of the insulator) than when field emission provides the seed electrons. As a result, criteria previously used to predict flashover discharges may not apply.

I. INTRODUCTION

Electrical flashover discharges across solid insulators in vacuum have been investigated due to the resultant loss of high voltage isolation and the damage which may occur.[1–5] A flashover discharge results from the random emission of a relatively small number of electrons at the cathode which is the precursor to an electron avalanche towards the anode across the surface of the dielectric [see Fig. 1(a)]. When using solid dielectrics for high voltage isolation, the source of seed electrons is usually electron emission from the cathode-vacuum-dielectric triple point.[6–9] Positive charging of the dielectric adjacent to the triple point by the seed electrons enhances the electric field there and helps sustain electron emission. As a result, flashover voltages often decrease as the thickness of the insulator decreases since the electric field enhancement, and hence electron emission, at the triple point increases. In applications where the surface or triple point are illuminated by UV radiation and the source of seed electrons is no longer dependent on the details of field emission from the triple point, the flashover voltage will not scale in the cited fashion. In fact, the flashover voltage may be lower by factors of 3–10 when the triple point or surface are illuminated by UV radiation than in the absence of irradiation.[10,11] The secondary emission and charging characteristics of the surface are equally as important in determining the flashover voltage as the initial emission mechanism for these conditions.[11] As we will show, the actual flashover voltage then depends on a tradeoff between enhanced electron field emission and a lower probability for electron multiplication when the dielectric thickness is decreased.

An often quoted criteria for initiating a surface flashover discharge in vacuum over a plane dielectric is that the secondary emission coefficient, for the electrons striking the dielectric, δ, must be greater than unity.[5,12] For these conditions, electron multiplication occurs as the electrons scatter across the surface, charging the surface, and desorbing gas. Electron multiplication, surface charging, and gas desorption are the precursors to other electron multiplication processes which perpetuate the discharge, such as gas phase ionization and further enhancement of the electric field at the triple point.[2,4,13] Most dielectrics possess this characteristic, $\delta \geqslant 1$, for some critical range of applied electric field. In complex geometries, as considered here, this simple criterion is difficult to apply since the orientation and value of the local electric field, and hence local secondary electron emission coefficient, are functions of position along the dielectric. In particular, the flashover voltage can depend on the angle of the dielectric with respect to the cathode and anode.[11] Another criterion, then, is required to characterize electron multiplication which accounts for the orientation of the electric field in a particular geometry, and for the past history of the surface.

In gas discharges, transport coefficients are used to characterize various plasma processes as a function of E/N (electric field/gas number density).[14] One such coefficient is the Townsend coefficient α (cm^{-1}) which is the characteristic distance for electron multiplication. α is defined by $n(z) = n(0) \times \exp(\alpha z)$, where $n(z)$ is the electron density at position z. Negative values of α denote net electron loss (e.g., recombination, attachment). It is desirable to use similar coefficients for surface discharges to obtain scaling parameters so that the results from one experiment may be applied to other conditions, and so avoid the ambiguity of simply quoting δ.

In this paper, we will use the results of a Monte Carlo simulation of surface charging to show that electron transport across solid dielectrics, and the precursor conditions to initiating a vacuum surface discharge, can be characterized by a coefficient similar to the Townsend coefficient used in gas discharges. The coefficient represents the electron multiplication and charging characteristics of the dielectric material in the chosen geometry as a function of its charging history. The surface Townsend coefficient is defined by

$$\frac{\text{Number of electrons collected at the anode}}{\text{Number of electrons released at the cathode}} = \exp(\alpha l), \tag{1}$$

where α is the Townsend coefficient of the insulator and l is

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the length between cathode and anode. Two values of α are defined; α, based on the total emission; and α_i, the instantaneous value. A negative Townsend coefficient indicates a net loss of electrons as they scatter across the surface, and this, instantaneously, denotes a nonflashover condition. A positive or zero Townsend coefficient indicates no electron loss or net electron multiplication and is the precursor condition for a flashover. We find that when the initial secondary electron emission does not depend upon field emission, $\alpha_i \geqslant 0$ is obtained at voltages significantly less than generally accepted as the flashover value.

In Sec. II, the Monte Carlo particle simulation is described followed by a discussion of surface charging in Sec. III. Results for the surface Townsend coefficient are presented in Sec. IV, and the tradeoff between field emission and having a low Townsend coefficient is discussed in Sec. V. Our concluding remarks are in Sec. VI.

II. MONTE CARLO SIMULATION FOR SURFACE CHARGING

A Monte Carlo particle simulation has been developed to model the scattering of electrons across the surface of a plane dielectric under high-voltage stress. The model uses as input the geometry and material properties of the electrodes and dielectric. In the simulation, we integrate the equation of motion of the electrons as they scatter from the dielectric while including secondary electron emission, backscatter, surface charging, and the deformation of the local electric field by surface charging. We consider the conditions when the seed electrons do not depend upon field emission from the triple point, as may occur when the cathode or dielectric are illuminated by UV radiation.[10,11]

The geometry used in this work is shown in Fig. 1(a). The calculation is performed in three dimensions. The geometry shown in Fig. 1(a) is a two-dimensional "slice" through the dielectric and is perpendicular to the surface of the dielectric over which the electrons scatter. The surface of the dielectric is modeled as 1 cm wide with periodic boundary conditions. Quartz ($\delta_{\max} = 2.4$ at normal incidence) is used as the dielectric. During the calculation, a primary particle is given a preassigned "weighting", representing a given number of electrons (typically 10^5 electrons per particle), and is released from the triple junction at a single point along the line of intersection of the dielectric and cathode. The equations of motion of the particle are integrated, based on the local electric field (see below), and the trajectory of the particle is updated. When a particle collides with the surface, the "weight" of the particle, w, is revised according to the backscatter yield δ_b for its energy and angle of incidence; $w \rightarrow \delta_b \times w$. A particle is added to the simulation at the site of the collision to represent secondary electron emission when $\delta > 0$. The weighting of the secondary particle is $\delta \times w$. An electrical charge of $-qw \times (1 - \delta_b - \delta)$ is deposited on the dielectric surface at the site of the collision adding to the local charge density $\rho(\mathbf{r})$. The secondary and backscatter yields as a function of energy and angle of incidence are as tabulated in the NASA NASCAP program.[15] A plot of secondary emission coefficient as a function of angle of incidence and energy appears in Fig. 2. As $\delta(\theta) \sim 1/\cos\theta$, where θ is measured from the normal, we restrict the value of δ to that corresponding to $\theta = 80°$ to account for microscopic surface roughness.

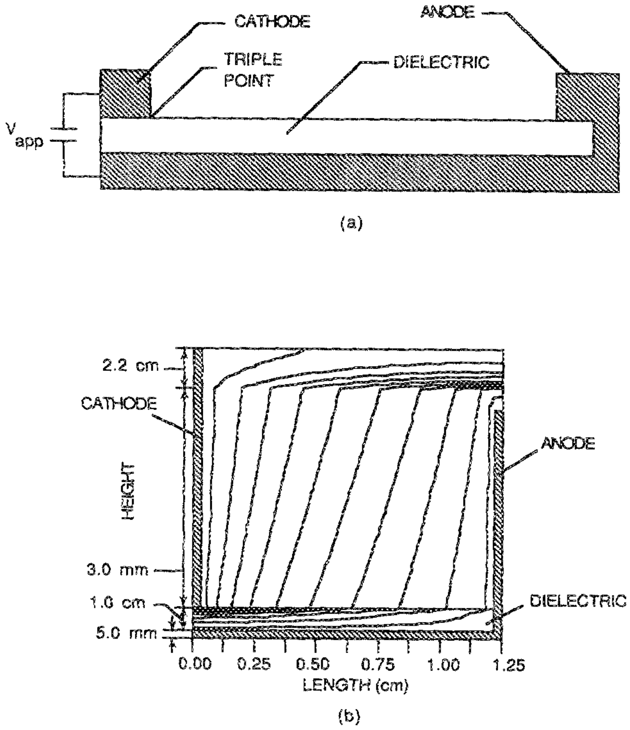

FIG. 1. Geometry used in this study. (a) Cathode and anode separated by a planar quartz dielectric surface. Electrons are emitted from the cathode triple point. (b) Equipotential lines for 1 cm dielectric thickness. The discontinuity in potential lines at the top of the figure is due to a change in vertical scale, as shown at the left. For this geometry the electric field is oriented more strongly into the dielectric as the thickness of the dielectric decreases.

The instantaneous electric field is given by

$$\mathbf{E}(\mathbf{r},t) = \mathbf{E}_0(\mathbf{r}) + \int \frac{\rho(\mathbf{r},t)(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3}\,dA', \qquad (2)$$

where $\mathbf{E}_0(\mathbf{r})$ is the local vacuum electric field, $\rho(\mathbf{r},t)$ is the instantaneous dielectric surface charge density, and the inte-

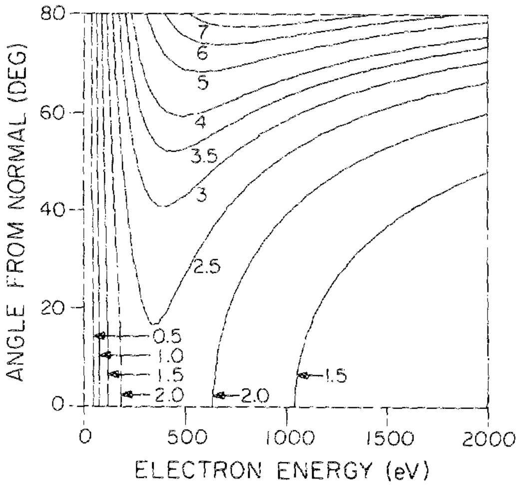

FIG. 2. Secondary emission coefficient for electrons scattering from quartz as computed from Ref. 15.

gral is over the surface of the dielectric. $\mathbf{E}_0(\mathbf{r})$ is calculated by solving Laplace's equation using the method of successive over-relaxation. The integral in Eq. (2) accounts for changes to the vacuum field in three dimensions due to charge left on the surface of the dielectric by electron scattering. The vacuum electric field $\mathbf{E}_0$ is shown in Fig. 1(b). (Note the change in vertical scales in the figure.) We find that the electrons remain within a few hundred microns of the surface as they scatter. That region is expanded in the figure.

III. IMPLICATIONS OF SURFACE CHARGING

A result of depositing charge on the dielectric by incident electrons having $\delta \neq 1$ is the formation of a sheath across the insulator surface. The sheath first forms near the cathode, where electrons initially strike the dielectric, and then spreads across the dielectric towards the anode. Immediately adjacent to the cathode, the surface charges positive. Away from the electrodes, the surface charge results in the sheath having a negative electric potential which is equal to the value which shields out the perpendicular component of the applied electric field within a few mm of the vacuum-dielectric interface. While the sheath is forming, the Townsend coefficient for electrons is negative since the source of charge on the dielectric is the emitted electrons. Following formation of the sheath at a particular location, electrons "pass" across the dielectric to a point of contact nearer to the anode in advance of the sheath, having on the average $\delta = 1$. By analogy, the formation of the sheath is equivalent to charging the capacitor formed by the cathode-dielectric-anode configuration. When the magnitude of released charge is sufficient for the sheath to cover the entire dielectric, the "capacitor" is fully charged. The magnitude of the local sheath potential at this time is such that the net secondary electron emission is unity. As a result, nearly the same number of electrons are collected at the anode as were emitted at the cathode, and the Townsend coefficient approaches zero.

Charging the dielectric and forming a sheath in this fashion are not sufficient by themselves to initiate a flashover discharge since the end result of the charging is to drive the Townsend coefficient towards zero, as discussed in Sec. IV. Having $\alpha_i = 0$ corresponds to there being no net electron multiplication. However having $\alpha_i = 0$ with a fully charged sheath implies that the insulating properties of the surface have been compromised since a conducting path of near constant resistance exists between cathode and anode. It also empirically corresponds to the onset of a surface flashover.[11] If $\alpha_i = 0$ is approached from negative values one never does have net electron multiplication. If, however, a geometry and charge voltage results in an initially positive Townsend coefficient, then $\alpha_i = 0$ is approached from positive values. In this case, there has been an electron avalanche which results in the rapid desorption of gas from the surface. The desorption of gas is the precursor to secondary electron processes (e.g., gas phase ionization) which can further sustain electron avalanche and lead to flashover.[2,4,13] Therefore, the positive surface Townsend coefficient is a precursor to flashover, though it in itself is not a sufficient condition.

Electron multiplication or depletion is governed by the values of the secondary yield and backscatter yield parameters for the surface, which are functions of both electron energy and angle of incidence of the electron. In general, the electron backscatter and secondary yields increase as the angle of incidence approaches grazing.[16] Therefore, the orientation of the electric field with respect to the dielectric is important in determining the net rate of electron production. For our model geometry, the orientation of the electric field is a function of the thickness of the dielectric [see Fig. 1(b)]. The electric field is oriented more strongly into the surface with thin dielectrics, which results in low secondary emission, as well as low transverse mobility.

IV. SURFACE TOWNSEND COEFFICIENTS

Surface Townsend coefficients for our model geometry calculated with the Monte Carlo particle simulation are plotted in Fig. 3 as a function of charge released from the triple point. The charge is released from the center of the cathode. We use the scaling parameter $\eta = \mathrm{C\ cm}^{-1}$ (Coulombs released/distance between the anode and cathode along the insulator surface). The values of α plotted in Fig. 3 are the integral values α corresponding to the total emission and collection of electrons. Therefore the condition $d\alpha/d\eta = 0$ corresponds to the instantaneous value α_i being zero. In Fig. 3(a), α is plotted for increasing anode-cathode voltage V_0 having fixed length between anode and cathode, l. We find that $\alpha_i = 0$ $(d\alpha/d\eta = 0)$ is approached at an equi-

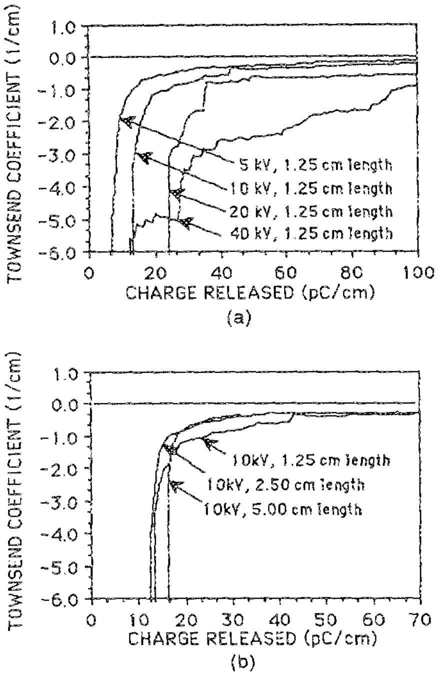

FIG. 3. Cumulative surface Townsend coefficient as a function of η (charge released from the triple point/distance between cathode and anode) for the geometry shown in Fig. 1 using a quartz dielectric of 3.5-mm thickness. The coefficients are for the integrated released and collected current. (a) Townsend coefficients for different cathode-anode voltage with fixed separation. (b) Townsend coefficients for fixed voltage and different cathode-anode separation.

librium charge that increases with increasing applied field. The initially negative values of α result from the dielectric in this geometry having, on the average, $\delta < 1$. As the surface charges and the sheath develop, α increases to reflect the fact that the average electron samples the surface less frequently, and the incident energy of the electron now corresponds more closely to that required for $\delta = 1$ due to the deceleration of the electron by the sheath. With higher applied field, the average electron is more energetic as it approaches the surface, and therefore more charge is required to obtain this deceleration as shown.

In Fig. 3(b), the cumulative α is plotted for fixed voltage with varying length. The equilibrium charge scales well with the normalized charge η for these conditions. The amount of released charge required to reach equilibrium increases proportional to the length of the insulator for fixed voltage, as more charge is required to form the sheath over a larger area. Equivalently the capacitance of the configuration increases with length [$C = (\epsilon w/d)\cdot l$, where w, l, and d are the width, the length, and thickness of the dielectric] and therefore more released electrons are required to charge the surface. The use of η is valid for linear scaling of a given geometry.

Our observation that $\alpha_i = 0$ is obtained at fixed η for constant charging voltage is consistent with the observation that flashover of UV illuminated surfaces occurs after a fixed incident fluence.[10,11] Our interpretation is that the fixed fluence corresponds to the photoemission of electrons sufficient to charge the dielectric to a condition where $\alpha_i \geqslant 0$ and which shields out the perpendicular component of the applied field. There is, then, a nearly one-to-one correspondence between η and fluence. The observations of Enlo and Gilgenbach[10,11] that flashover occurs when the applied field is shielded by the sheath corresponding to $\alpha_i = 0$ confirms that this condition is a precursor to flashover.

As negative charge is deposited on the dielectric near the cathode, transverse components of the electric field are generated which may be comparable ($>$ kV/cm) to the longitudinal field at the close approach (l00's μm) of the scattered electrons to the charge. These space charge fields cause the electrons scattering across the dielectric to spread laterally, as shown in the plot of negative surface charge in Fig. 4. The negative charge continues to spread laterally until the sheath covers the surface. The areas devoid of negative charge adjacent to the electrodes are actually charged positive as has been predicted by other investigators.[2,4,5,16,17] The effect of positive charge buildup at the cathode is to enhance the applied field, and consequently induce a greater number of particles per unit time to be released from the cathode. This induced emission from the triple point is also a precursor to flashover.

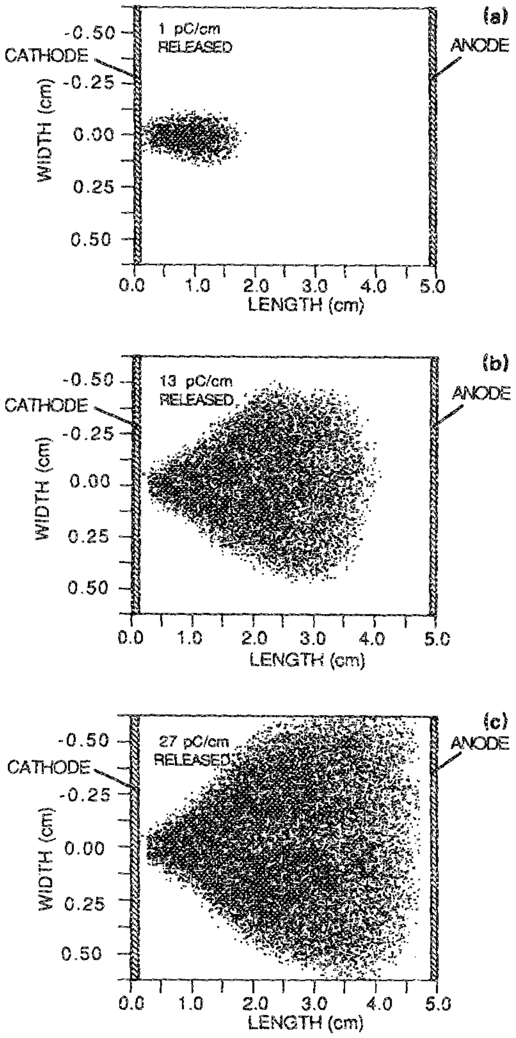

FIG. 4. Location of negative charge as a function of released charged from the triple point for conditions where $\alpha < 0$. The view is looking down on the dielectric with the cathode at left and anode at right. The apparently uncharged regions adjacent to the cathode and anode in (c) actually have positive surface charge.

V. TRADEOFF IN DIELECTRIC THICKNESS WITH RESPECT TO FIELD EMISSION

For our geometry, the electric field is oriented less strongly into the dielectric surface with thicker dielectrics. As a result, the average electron striking the surface does so with more energy and the secondary electron coefficients tend to be higher because of the advantageous scaling with grazing angle of incidence. The critical value of vacuum applied field for which the instantaneous and cumulative surface Townsend coefficients are positive, and $\alpha_i = 0$ is approached from positive values, therefore decreases with increasing dielectric thickness. The cumulative and instantaneous Townsend coefficents for such conditions are plotted in Fig. 5.

Accordingly, there is a trade-off between the magnitude of the applied electric field and the dielectric insulator thickness with respect to preventing a positive Townsend coefficient. This is shown by a parametric study of applied voltage and dielectric thicknesses (0–10 cm), summarized in Fig. 6. The parameter space can be divided into regions where α_i is always $\leqslant 0$ (lower voltages for a given dielectric thickness) and where $\alpha_i > 0$ either instantaneously or cumulatively. Provided that electric field emission is not a necessary source of seed electrons as in a UV irradiated environment, thinner dielectrics are more likely to approach $\alpha_i = 0$ from negative values; thicker dielectrics are more likely to approach $\alpha_i = 0$

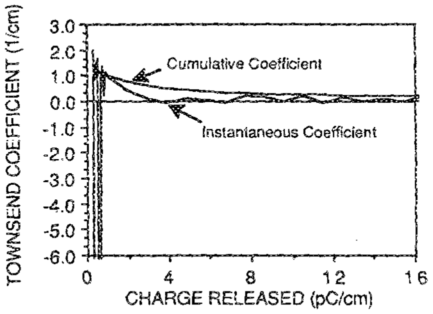

FIG. 5. Instantaneous and cumulative surface Townsend coefficients where $\alpha \geqslant 0$. The dielectric thickness is 1.5 cm. The Townsend coefficient builds from a negative value to a positive value as the released charge initiates an avalanche that eventually extends all the way to the anode. Obtaining a positive Townsend coefficient is a precursor to flashover.

from positive values. The intermediate regime shown in Fig. 6 corresponds to there being high "shot to shot" variation in the charging. The end product of α_i approaching zero from either positive or negative values is there being no net electron multiplication. We propose, though, that the conditions where α_i approaches zero from positive values is more prone to flashover due to the more intense positive field enhancement at the triple point and more rapid rate of gas desorption. We would then predict flashover voltages as low as 3–10 kV/cm for quartz under UV illumination, compared to 30–40 kV/cm for nonilluminated surfaces.[1,2]

Distortion of the local electric field by the sharp edges of the cathode and the discontinuity in dielectric constant at the triple point can produce local field enhancement of orders of magnitude. This may be sufficiently high to promote electric field emission of electrons. For our conditions, field enhancement and the probability for field emission increase as the thickness of the dielectric decreases. The electric field at the triple point, normalized by V_0/l (where V_0 is the vacuum applied voltage and l is the anode-cathode separation) is plotted as a function of dielectric thickness in Fig. 7.

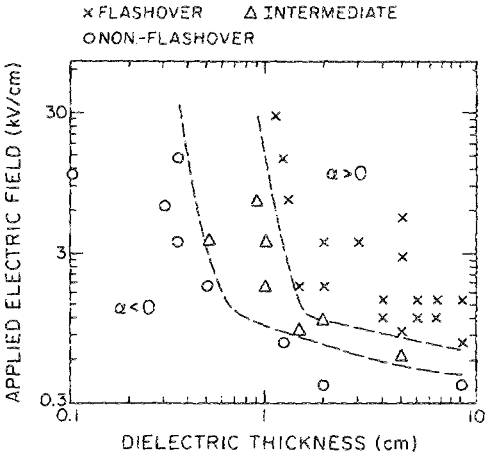

FIG. 6. Regions of negative and positive surface Townsend coefficient in the voltage-dielectric thickness plane for the geometry shown in Fig. 1. There is a trade-off between the magnitude of applied electric field and the thickness of the dielectric with respect to insuring that $\alpha_i \leqslant 0$. The intermediate region corresponds to having high shot to shot variation.

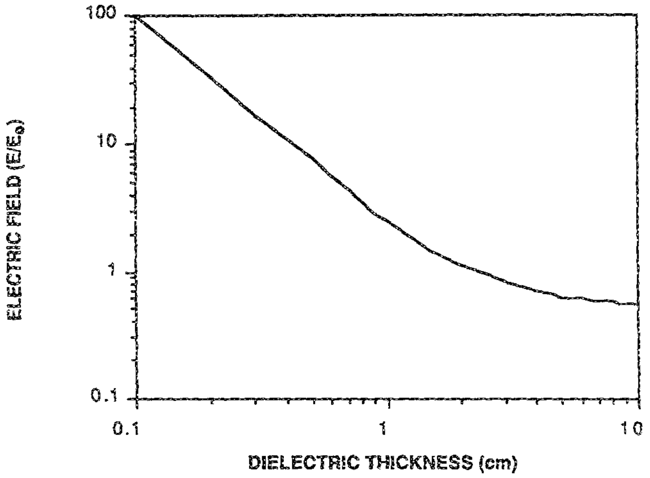

FIG. 7. Electric field at the triple point (normalized by $E_0 = V_0/l$ where V_0 is the vacuum applied voltage and l is the anode-cathode separation) as a function of dielectric thickness for the geometry shown in Fig. 1. For our conditions, field enhancement and the probability for field emission, increase as the thicknesss of the dielectric decreases.

As it is generally accepted that field emission occurs at field values of 500 kV cm^{-1}–1 MV cm^{-1}, moderate values of V_0/l with thin dielectrics may be sufficient to cause field emission.

As discussed above, thinner dielectrics in our geometry are less prone to rapidly obtaining $\alpha_i \geqslant 0$ and therefore by implication less susceptible to flashover discharges provided that field emission is the required source of seed electrons. However, thinner dielectrics maximize the probability for field emission. There is then, a trade-off in dielectric thickness (or orientation of the electric field) with respect to flashover voltage when considering field emission and surface charging.

VI. CONCLUDING REMARKS

A model has been developed for charging of solid plane dielectrics under high voltage stress which predicts the precursor conditions for a surface flashover discharge in the presence of UV radiation. The insulating strength of the dielectric is described in terms of an effective Townsend coefficient, α, and scaling parameter η (Coulombs released/length of dielectric). The instantaneous surface Townsend coefficient α_i approaches zero as η increases. A value of $\alpha_i \geqslant 0$ corresponds to conditions which are the precursors to flashover discharges.[11] A trade-off exists between the flashover voltage and the thickness of the dielectric insulator. In the absence of field emission thicker dielectrics require a lower applied electric field to cause α_i to be $\geqslant 0$. Thin dielectrics, though, are more prone to field emission at the triple point. With UV illumination, voltages resulting in $\alpha_i \geqslant 0$ (or $d\alpha/d\eta = 0$), are significantly lower than the flashover voltages measured in the absence of illumination. These results agree with experiments[10,11] and imply that obtaining high flashover voltages (exceeding many 10's of kV cm^{-1}) depends on the suppression of the generation of seed electrons by illumination of the triple point or dielectric, or a reduction in the rate of charging of the dielectric.

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